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Numerical Methods - 1

Numerical methods for IVP.

consider a d.eqn of first order and first degree in the form $\frac{dy}{dx} = f(x, y)$ with the initial codn $y(x_0) = y_0$, that is $y = y_0$ at $x = x_0$. This problem of finding y is called an IVP.

Taylor's Series method

Consider the IVP : $\frac{dy}{dx} = f(x, y)$ and $y(x_0) = y_0$.
The sol^n $y(x)$ is approximated to a power series in $(x - x_0)$ using Taylor's Series. Then we can find the value of y for various values of x in nbd of x_0.

We have Taylor's Series expansion of $y(x)$ about the point x_0 in the form

$$y(x) = y(x_0) + (x - x_0) y'(x_0) + \frac{(x - x_0)^2}{2!} y''(x_0) + \frac{(x - x_0)^3}{3!} y'''(x_0) + \cdots$$

here $y'(x_0), y''(x_0), \cdots$ denote the value of the derivatives $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \cdots$ at x_0 which can be found by making use of the data.

Problems

1) Use Taylor's series method to find y at $x = 0.1, 0.2, 0.3$ considering terms upto the third degree given that $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$

>> Taylor's series exp^n of $y(x)$ is given by

$$y(x) = y(x_0) + (x - x_0) y'(x_0) + \frac{(x - x_0)^2}{2!} y''(x_0) + \frac{(x - x_0)^3}{3!} y'''(x_0) + \cdots$$

By data $x_0 = 0$, $y_0 = 1$ and

$$\frac{dy}{dx} = x^2 + y^2$$

$$\Rightarrow y' = x^2 + y^2$$

$\therefore\ y(x) = y(0) + x\,y'(0) + \frac{x^2}{2!} y''(0) + \frac{x^3}{3!} y'''(0) + \text{---}$ (1)

we need to calculate $y'(0),\ y''(0),\ y'''(0)$

consider $y' = x^2 + y^2 \quad \therefore y'(0) = x_0^2 + y_0^2$

$y'(0) = 0^2 + 1^2 = 1$

Diff wrt x

$y'' = 2x + 2yy' \quad \therefore y''(0) = 2x_0 + 2y_0 y'(0)$

$y''(0) = 2(0) + 2(1)(1) = 2$

again wrt x

$y''' = 2x + 2[yy'' + y'y']$

$y''' = 2 + 2yy'' + 2y'^2 \quad \therefore y'''(0) = 2 + 2y_0 y''(0) + 2(y'(0))^2$

$y'''(0) = 2(1) + 2(1)(2) + 2(1)^2$

$y'''(0) = 2 + 4 + 2 = 6 + 2 = 8$

(1) $\Rightarrow$

$y(x) = 1 + x(1) + \frac{x^2}{2}(2) + \frac{x^3}{6} \cdot 8$

$\boxed{y(x) = 1 + x + x^2 + \frac{4x^3}{3}}$ is called Taylor's Series appm upto the 3rd degree

we need to put $x = 0.1,\ 0.2,\ 0.3$

$y(0.1) = 1 + (0.1) + (0.1)^2 + \frac{4(0.1)^3}{3} = 1.1113$

$y(0.2) = 1 + (0.2) + (0.2)^2 + \frac{4(0.2)^3}{3} = 1.2507$

$y(0.3) = 1 + 0.3 + (0.3)^2 + \frac{4(0.3)^3}{3} = 1.426$

2) Find y at $x = 1.02$ correct to five decimal places given $dy = (xy-1)\,dx$ and $y = 2$ at $x = 1$ applying Taylor's Series method. (11)

>> Taylor's Series expn is given by.

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \cdots$$

By data $x_0 = 1$, $y_0 = 2$ and

$$y' = \frac{dy}{dx} = xy - 1$$

Since the no of derivatives for approximation is not specifically mentioned, we shall have the apprn upto 3rd degree.

$$y(x) = y(1) + (x-\cancel{1})y'(1) + \frac{(x-1)^2}{2!}y''(1) + \frac{(x-1)^3}{3!}y'''(1) \quad \text{--- (1)}$$

Consider

$y' = xy - 1$; $y'(1) = x_0 y_0 - 1 = 1(2) - 1 = 1$

$y'' = xy' + y$; $y''(1) = x_0 y'(1) + y_0 = 1(1) + 2 = 3$

$y''' = xy'' + y' + y'$

$\Rightarrow y''' = xy'' + 2y'$; $y'''(1) = x_0 y''(1) + 2y'(1)$

$= 1(3) + 2(1) = 3 + 2 = 5$

$y^{IV} = xy''' + y'' + 2y''$

$\Rightarrow y^{IV} = xy''' + 3y''$; $y^{IV}(1) = x_0 y'''(1) + 3y''(1)$

$= 1\cdot(5) + 3(3)$

$= 5 + 9 = \underline{14}$

(1) $\Rightarrow$

$$y(x) = 2 + (x-1)(1) + \frac{(x-1)^2}{2}(3) + \frac{(x-1)^3}{6}5 + \frac{(x-1)^4}{24}14$$

$$y(1.02) = 2 + (1.02-1) + \frac{(1.02-1)^2}{2}3 + \frac{(1.02-1)^3}{6}5 + \text{—}$$

$$y(1.02) = \underline{\underline{2.02061}}.$$

upto 3rd term only + const

3) From Taylor's series method, find $y(0.1)$ considering upto 4th degree term of $y(x)$ satisfies the eqn $\frac{dy}{dx} = x - y^2$, $y(0) = 1$.

>> Taylor's series expn is given by

$$y(x) = y(x_0) + (x - x_0) y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \cdots$$

By data $y_0 = 1$, $x_0 = 0$.

$$y' = x - y^2$$

$$\therefore y(x) = y(0) + x y'(0) + \frac{x^2}{2!} y''(0) + \frac{x^3}{3!} y'''(0) + \frac{x^4}{4!} y^{iv}(0) \quad \text{--- (1)}$$

Consider $y' = x - y^2$ $\qquad \therefore y'(0) = x_0 - y_0^2 = 0 - 1^2 = -1$

$y'' = 1 - 2yy'$ $\qquad y''(0) = 1 - 2y_0 y'(0) = 1 - 2.1.(-1)$

$y''(0) = 3$

$y''' = 0 - 2[yy'' + (y')^2]$

$y''' = -2[yy'' + y'^2]$ $\qquad y'''(0) = -2[(1)(3) + (-1)^2]$

$= -6 - 2 = -8$

$y^{iv} = -2[yy''' + y'y'' + 2y'y'']$

$= -2[yy''' + 3y'y'']$ $\qquad \therefore y^{iv}(0) = -2[1(-8) + 3(-1)(3)]$

$= -2[-8 - 9]$

$= -2[-17]$

$= 34$

(1) $\Rightarrow$

$$y(x) = 1 + x(-1) + \frac{x^2}{2}(3) + \frac{x^3}{6}(-8) + \frac{x^4}{24}(34)$$

$$y(x) = 1 + (0.1)(-1) + \frac{(0.1)^2}{2}(3) + \frac{(0.1)^3}{6}(-8) + \frac{(0.1)^4}{24}(34)$$

$$y(x) = 0.9138$$

4) Use Taylor's series method to obtain a power series in $(x-4)$ for the Eq^n $5x\frac{dy}{dx}+y^2-2=0$, $x_0=4$, $y_0=1$ and use it to find y at $x=4.1, 4.2, 4.3$ correct to four decimal places. (12)

>> Taylor's series exp^n is given by

$$y(x)=y(x_0)+(x-x_0)y'(x_0)+\frac{(x-x_0)^2}{2!}y''(x_0)+\text{------}$$

Since $x_0=4$, $y_0=1$ and

$$y(x)=y(4)+(x-4)y'(4)+\frac{(x-4)^2}{2}y''(4)+\text{------}$$

consider $5xy'+y^2-2=0$

$\therefore 5x_0y_0'+y_0^2-2=0 \Rightarrow 5(4)y'+1-2=0$

$20y'+1-2=0$

$20y'-1=0$

$20y'=1 \Rightarrow y'(4)=\frac{.1}{20}$

$y'(4)=\frac{1}{20}=\underline{0.05}$

(1) Diff w.r.t x

$5[xy''+y']+2yy'=0$

$5\left[(4)\cdot y''+\frac{1}{20}\right]+2\cdot(1)\cdot\frac{1}{20}=0$

$20y''+\frac{5}{20}+\frac{2}{20}=0$

$20y''+\frac{1}{4}+\frac{1}{10}=0$

$\frac{1}{4}+\frac{1}{10}=0$

$\frac{10+4}{40}=\frac{14}{40}$

$\frac{7}{20}$

$20y''+\frac{7}{20}=0$

$20y''=-\frac{7}{20}$

$y''=-\frac{7}{20}\cdot\frac{1}{20}$

$y''=-\frac{7}{40}=\underline{-0.175}$

5) Use Taylor's series method to find $y(4.1)$ given that $\frac{dy}{dx} = \frac{1}{x^2+y}$ and $y(4)=4$. (13)

>> Taylor's series expⁿ is given by

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \ldots$$

By data $y' = \frac{1}{x^2+y}$ and $y_0 = 4$ at $x_0 = 4$

$$\therefore y(x) = y(4) + (x-4)y'(4) + \frac{(x-4)^2}{2!}y''(4) + \frac{(x-4)^3}{3!}y'''(4) \quad \text{--- (1)}$$

given $y' = \frac{1}{x^2+y}$

(or) $y'(x^2+y) = 1$ --- (2) $\therefore$

Sub initial values $y'(4)(4^2+4) = 1$

$y'(4)(16+4) = 1$

$y'(4)(20) = 1$

$$\boxed{y'(4) = \frac{1}{20}} \boxed{= 0.05}$$

(2) Diff wrt x

$y'(2x+y') + (x^2+y)y'' = 0$

Sub initial values

$0.05(2(4)+0.05) + (16+4)\cdot y''(4) = 0$

$0.05(8+0.05) + (20)y''(4) = 0$

$0.05(8.05) + 20y''(4) = 0$

$0.4025 + 20y''(4) = 0$

$20y''(4) = -0.4025$

$$\boxed{y''(4) = -\frac{0.4025}{20} = -0.020125}$$

we observe that the value of the derivatives are small enough and the third degree term can also be neglected. Substituting these values in (1) for computing $y(4.1)$

$y(4.1) = 4 + (4.1-4)(0.05) + \frac{(4.1-4)^2}{2}(-0.020125)$

$\boxed{y(4.1) = 4.0049}$

6) Use Taylor's series method to solve $y' = x^2 + y$ in the range $0 \le x \le 0.2$ by taking step size $h = 0.1$ given that $y = 10$ at $x = 0$ initially considering terms upto the fourth degree.

>> In this problem, since the step size is specified as 0.1 the problem has to be done in two stages, we've to first find $y(0.1)$ and use this as the initial cond^n to compute $y(0.2)$

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \frac{(x-x_0)^4}{4!}y^{iv}(x_0) + \cdots \quad \text{--- (1)}$$

<u>I stage</u> : By data

$y' = x^2 + y$, $x_0 = 0$, $y_0 = 10$

(1) =>

$$y(x) = y(0) + x y'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y^{iv}(0) \quad \text{--- (2)}$$

$y' = x^2 + y$

$\Rightarrow y'(0) = 0^2 + 10 = 10 \Rightarrow \boxed{y'(0) = 10}$

Diff wrt x

$y'' = 2x + y'$; $y''(0) = 2(0) + 10 = 10 \Rightarrow \boxed{y''(0) = 10}$

Diff wrt x

$y''' = 2 + y''$; $y'''(0) = 2 + 10 = 12 \Rightarrow \boxed{y'''(0) = 12}$

Diff wrt x

$y^{iv} = y'''$; $\boxed{y^{iv} = 12}$

7) Employ Taylor's series method to find y at $x=0.1$ and 0.2 correct to four places of decimal in step size of 0.1 given the linear d.eqⁿ $\frac{dy}{dx} - 2y = 3e^x$ whose solⁿ passes through the origin.

→ By data $y' = 2y + 3e^x$ and $y(0)=0$, ie $y_0 = 0$ and $x_0 = 0$

Taylor's series expⁿ is given by

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \cdots \quad \text{---} (1)$$

Step 1: We shall compute $y(0.1)$

consider $y' = 2y + 3e^x$; $y'(0) = 2(0) + 3e^0 = 3$

$y'' = 2y' + 3e^x$; $y''(0) = 2(3) + 3 = 9$

$y''' = 2y'' + 3e^x$; $y'''(0) = 2(9) + 3 = 21$

(1) ⟹

$$y(0.1) = y(0) + (0.1)y'(0) + \frac{(0.1)^2}{2}y''(0) + \frac{(0.1)^3}{6}y'''(0)$$

$$y(0.1) = 0 + (0.1)3 + \frac{0.01}{2}(9) + \frac{0.001}{6}(21)$$

$$\boxed{y(0.1) = 0.3485}$$

Step 2: $y(0.2)$

$y' = 2y + 3e^x$ $x_0 = 0.1$, $y_0 = 0.3485$

$y'(0) = 2y(0.1) + 3e^{0.1}$; $y'(0.1) = 4.0125$

$y'' = 2y' + 3e^x$; $y''(0.1) = 2(4.0125) + 3e^{0.1} = 11.3405$

$y''' = 2y'' + 3e^x$; $y'''(0.1) = 2(11.3405) + 3e^{0.1} = 25.9965$

∴ (1) ⟹

$y(0.2) = 0.8108$

with $x = 0.1$ and $x_0 = 0$ (2) becomes.

$$y(0.1) = y(0) + (0.1)y'(0) + \frac{(0.1)^2}{2}y''(0) + \frac{(0.1)^3}{6}y'''(0) + \frac{(0.1)^4}{24}y^{IV}(0)$$

$$y(0.1) = 10 + 0.1(10) + \frac{(0.1)^2}{2}(10) + \frac{(0.1)^3}{6}(12) + \frac{(0.1)^4}{24}(12)$$

$$\boxed{y(0.1) = 11.05205 \simeq 11.0521}$$

II. Stage : Now taking $x_0 = 0.1$ and $y_0 = 11.0521$

$y' = x^2 + y$; $y'(0.1) = (0.1)^2 + 11.0521 = 11.0621$

$y'' = 2x + y'$; $y''(0.1) = 2(0.1) + 11.0621 = 11.2621$

$y''' = 2 + y''$; $y'''(0.1) = 2 + 11.2621 = 13.2621$

$y^{IV} = y'''$; $y^{IV}(0.1) = 13.2621$

with $x = 0.2$ and $x_0 = 0.1$ (2) $\Rightarrow$

$$y(0.2) = y(0.1) + (0.1)y'(0.1) + \frac{(0.2-0.1)^2}{2}y''(0.1) + \frac{(0.2-0.1)^3}{3!}y'''(0.1) + \frac{(0.2-0.1)^4}{4!}y^{IV}(0.1)$$

$$y(0.2) = y(0.1) + (0.1)y'(0.1) + \frac{(0.1)^2}{2}y''(0.1) + \frac{(0.1)^3}{6}y'''(0.1) + \frac{(0.1)^4}{24}y^{IV}(0.1)$$

$$y(0.2) = 11.0521 + 0.1(11.0621) + \frac{0.01}{2}(11.2621) + \frac{0.001}{6}(13.2621) + \frac{0.0001}{24}(13.2621)$$

$$y(0.2) = 12.216776 \simeq 12.2168$$

Modified Euler's Method.

Consider the IVP $\frac{dy}{dx} = f(x, y)$; $y(x_0) = y_0$.

we need to find y at $x_1 = x_0 + h$.

we first obtain $y(x_1) = y_1$ by applying Euler's formula and this value is regarded as the initial apprn for y_1 usually denoted by $y_1^{(0)}$ also called as the predicted value of y_1.

Euler's formula is given by

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$

Since the accuracy is poor in this formula this value y_1 is successively improved (corrected) to the desired degree of accuracy by the follng Modified Euler's formula. where the successive apprn are denoted by $y_1^{(1)}, y_1^{(2)}, y_1^{(3)}, \ldots$ etc

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

Each of the succeeding apprn is better than the preceding ones, They are called corrected values. Euler's formula and modified Euler's formula jointly we are also called as Euler's predictor and corrector formulae,

1) Given $\frac{dy}{dx} = 1 + \frac{y}{x}$, $y=2$ at $x=1$, find approximate value of y at $x=1.4$ by taking step size $h=0.2$ applying modified Euler's method. Also find the value of y at $x=1.2$ and $x=1.4$ from the analytical sol^n of the eq^n?

>> The problem has to be worked in two stages.

<u>I stage</u> : $x_0 = 1$, $y_0 = 2$

$f(x,y) = 1 + y/x$, $h = 0.2$

$x_1 = x_0 + h = 1 + 0.2 = 1.2$

$\boxed{x_1 = 1.2}$

$y(x_1) = y_1$

$\boxed{y(1.2) = ?}$

Now $f(x_0, y_0) = 1 + y_0/x_0 = 1 + \frac{2}{1} = 1 + 2 = 3$

we have Euler's formula

$$y_1^{(0)} = y_0 + h f(x_0, y_0) \quad \text{——— (1)}$$

$$\therefore y_1^{(0)} = 2 + (0.2)(3)$$

$$\boxed{y_1^{(0)} = 2.6}$$

further we have modified Euler's formula

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right] \quad \text{——— (2)}$$

$$= 2 + \frac{0.2}{2}\left[3 + \left(1 + y_1^{(0)}/x_1\right)\right]$$

$$= 2 + 0.1\left[3 + \left(1 + 2.6/1.2\right)\right]$$

$$\boxed{y_1^{(1)} = 2.6167}$$

Next approx $y_1^{(2)}$ is got just by replacing the value of $y_1^{(1)}$ in place of $y_1^{(0)}$ Now

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$y_1^{(2)} = 2 + 0.1\left[3 + 1 + y_1^{(1)}/x_1\right]$$

$$= 2 + 0.1\left[4 + \frac{2.6167}{1.2}\right]$$

$$= 2 + 0.1\left[4 + 2.1806\right]$$

$$\boxed{y_1^{(2)} = 2.6181}$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 2 + 0.1\left[3 + 1 + \frac{y_1^{(2)}}{x_1}\right]$$

$$= 2 + 0.1\left[4 + \frac{2.6181}{1.2}\right]$$

$$= 2 + 0.1\left[4 + 2.18175\right]$$

$$\boxed{y_1^{(3)} = 2.6182}$$

$$y_1^{(4)} = y_0 + \frac{h}{2}\left[f(x_0 y_0) + f(x_1, y_1^{(3)})\right]$$

$$= 2 + 0.1\left[3 + 1 + \frac{y_1^{(3)}}{x_1}\right]$$

$$= 2 + 0.1\left[4 + \frac{2.6182}{1.2}\right]$$

$$\boxed{y_1^{(4)} = 2.6182}$$

II Stage: we repeat the process by taking $y(1.2) = 2.6182$ as the initial cond^n.

$x_0 = 1.2$, $y_0 = 2.6182$

$f(x_0, y_0) = 1 + y_0/x_0 = 1 + \frac{2.6182}{1.2} = 1 + 2.1818 = \underline{3.1818}$

$\therefore \boxed{f(x_0, y_0) = 3.1818}$

$x_1 = x_0 + h = 1.2 + 0.2 = 1.4$

$\therefore \boxed{x_1 = 1.4}$ $\boxed{y(x_1) = y_1 = y(1.4) = ?}$

from ①

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$y_1^{(0)} = 2.6182 + (0.2)(3.1818)$

$\boxed{y_1^{(0)} = 3.2546}$

from ②

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$

$y_1^{(1)} = 2.6182 + \frac{0.2}{2}\left[3.1818 + 1 + \frac{y_1^{(0)}}{x_1}\right]$

$= 2.6182 + 0.1\left[3.1818 + 1 + \frac{3.2546}{1.4}\right]$

$= 2.6182 + 0.1\left[4.1818 + 2.3247\right]$

$\boxed{y_1^{(1)} = 3.2689}$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 2.6182 + 0.1\left[3.1818 + 1 + \frac{y_1^{(1)}}{x_1}\right]$$

$$= 2.6182 + 0.1\left[4.1818 + \frac{3.2689}{1.4}\right]$$

$$\boxed{y_1^{(2)} = 3.2699}$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 2.6182 + 0.1\left[3.1818 + 1 + \frac{y_1^{(2)}}{x_1}\right]$$

$$= 2.6182 + 0.1\left[4.1818 + \frac{3.2699}{1.4}\right]$$

$$= 2.6182 + 0.1\left[4.1818 + 2.3356\right]$$

$$\boxed{y_1^{(3)} = 3.2699}$$

Thus $\boxed{\begin{matrix} y(1.2) = 2.6182 \\ y(1.4) = 3.2699 \end{matrix}}$ is the Required accuracy
and

2) Using modified Euler's method find y at $x = 0.2$ given $\frac{dy}{dx} = 3x + \frac{1}{2}y$ with $y(0) = 1$ taking $h = 0.1$. perform 3 iterations at each step.

>> We need to find $y(0.2)$ by taking $h = 0.1$

This implies that the problem has to be done in two stages.

I stage By data $x_0 = 0,\ y_0 = 1,\ h = 0.1$

$$f(x, y) = 3x + (y/2)$$

$$f(x_0, y_0) = 3x_0 + y_0/2 = 0 + 1/2 = 0.5$$

$$\boxed{f(x_0, y_0) = 0.5}$$

$$x_1 = x_0 + h = 0 + 0.1$$

$$\boxed{x_1 = 0.1}$$

$$y(x_1) = y_1 = y(0.1) = ?$$

From Euler's formula:

$y_1^{(0)} = y_0 + h f(x_0, y_0)$ we obtain

$$y_1^{(0)} = 1 + 0.1(0.5) = 1.05$$

$$\therefore \boxed{y_1^{(0)} = 1.05}$$

we have modified Euler's formula

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$ 1st iteration

$$= 1 + \frac{0.1}{2}\left[0.5 + 3x_1 + \frac{y_1^{(0)}}{2}\right]$$

$$= 1 + 0.05\left[0.5 + 3(0.1) + \frac{1.05}{2}\right]$$

$$= 1 + 0.05\left[0.5 + 0.3 + 0.525\right]$$

$$= 1 + 0.05\left[1.325\right]$$

$$\boxed{y_1^{(1)} = 1.0663}$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1 + 0.05\left[0.5 + 3x_1 + \frac{y_1^{(1)}}{2}\right]$$

$$= 1 + 0.05\left[0.5 + 3(0.1) + \frac{1.0663}{2}\right]$$

$$\boxed{y_1^{(2)} = 1.0667}$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1 + 0.05\left[0.5 + 3x_1 + \frac{y_1^{(2)}}{2}\right]$$

$$= 1 + 0.05\left[0.5 + 3(0.1) + \frac{1.0667}{2}\right]$$

$$\boxed{y_1^{(3)} = 1.0667}$$

Thus ** $\boxed{y(0.1) = 1.0667}$ **

II Stage Now ; let

$$x_0 = 0.1, \quad y_0 = 1.0667$$

we've $f(x, y) = 3x + y/2$

$$\therefore f(x_0, y_0) = 3x_0 + y_0/2 = 3(0.1) + \frac{1.0667}{2}$$

$$\boxed{f(x_0, y_0) = 0.8334}$$

$$x_1 = x_0 + h = 0.1 + 0.1 = 0.2$$

$$\boxed{x_1 = 0.2}$$

$$y_1 = (x_1) = y(0.2) = ?$$

Euler's formula we obtain

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$

$$= \cancel{0.}\ 1.0667 + 0.1(0.8334) = 1.15$$

$$\boxed{y_1^{(0)} = 1.15}$$

Next from modified Euler's formula.

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 1.0667 + \frac{0.1}{2}\left[0.8334 + 3x_1 + \frac{y_1^{(0)}}{2}\right]$$

$$= 1.0667 + 0.05\left[0.8334 + 3(0.2) + \frac{1.15}{2}\right]$$

$$\boxed{y_1^{(1)} = 1.1671}$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1.0667 + 0.05\left[0.8334 + 3x_1 + \frac{y_1^{(1)}}{2}\right]$$

$$= 1.0667 + 0.05\left[0.8334 + 3(0.2) + \frac{1.1671}{2}\right]$$

$$\boxed{y_1^{(2)} = 1.1675}$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1.0667 + 0.05\left[0.8334 + 3x_1 + \frac{y_1^{(2)}}{2}\right]$$

$$= 1.0667 + 0.05\left[0.8334 + 3(0.2) + \frac{1.1675}{2}\right]$$

$$\boxed{y_1^{(3)} = 1.1676}$$

Thus $y(0.2) = 1.1676$

3) using modified Euler's method find $y(0.2)$ correct to four decimal places solving the eqn $\frac{dy}{dx} = x - y^2$, $y(0) = 1$ taking $h = 0.1$ (19) 5

>> We shall first compute $y(0.1)$ and use this value to compute $y(0.2)$

I stage: By data $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$f(x, y) = x - y^2$

$\therefore f(x_0, y_0) = x_0 - y_0^2 = 0 - 1^2 = -1$

$\therefore \boxed{f(x_0, y_0) = -1}$

$x_1 = x_0 + h = 0 + 0.1$

$\boxed{x_1 = 0.1}$ $\boxed{y(x_1) = y_1 = y(0.1) = ?}$

From Euler's formula

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$y_1^{(0)} = 1 + 0.1(-1) = 0.9$

$\boxed{y_1^{(0)} = 0.9}$

we've modified Euler's formula

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1 + \frac{0.1}{2}[-1 + x_1 - \{y_1^{(0)}\}^2]$

$= 1 + 0.05[-1 + 0.1 - (0.9)^2]$

$= 1 + 0.05[-1 + 0.1 - 0.81]$

$\boxed{y_1^{(1)} = 0.9145}$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$
$$= 1 + 0.05\left[-1 + x_1 - (y_1^{(1)})^2\right]$$
$$= 1 + 0.05\left[-1 + 0.1 - (0.9145)^2\right]$$
$$= 1 + 0.05\left[-1 + 0.1 - 0.8363\right]$$

$$\boxed{y_1^{(2)} = 0.9132}$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$
$$= 1 + 0.05\left[-1 + x_1 - (y_1^{(2)})^2\right]$$
$$= 1 + 0.05\left[-1 + 0.1 - (0.9132)^2\right]$$
$$= 1 + 0.05\left[-1 + 0.1 - 0.8339\right]$$

$$\boxed{y_1^{(3)} = 0.9133}$$

$$y_1^{(4)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(3)})\right]$$
$$= 1 + 0.05\left[-1 + x_1 - (y_1^{(3)})^2\right]$$
$$= 1 + 0.05\left[-1 + 0.1 - (0.9133)^2\right]$$
$$= 1 + 0.05\left[-1 + 0.1 - 0.8341\right]$$

$$\boxed{y_1^{(4)} = 0.9133}$$

** Thus $\boxed{y(0.1) = 0.9133}$ **

<u>II Stage</u> Now, let $x_0 = 0.1$, $y_0 = 0.9133$

$f(x,y) = x - y^2$

$f(x_0, y_0) = x_0 - y_0^2 = (0.1) - (0.9133)^2$

$= 0.1 - 0.8341$

*** $\boxed{f(x_0, y_0) = -0.7341}$ ***

$x_1 = x_0 + h = 0.1 + 0.1 = 0.2$

$\boxed{x_1 = 0.2}$ $\quad y(x_1) = \underline{y(0.2) = ?}$

Now from the modified Euler's formula,

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$

Euler's formula $y_1^{(0)} = y_0 + h f(x_0, y_0) = 0.9133 + 0.1(-0.7341)$

$\boxed{y_1^{(0)} = 0.8399}$

$\therefore y_1^{(1)} = 0.9133 + \frac{0.1}{2}\left[-0.7341 + x_1 - (y_1^{(0)})^2\right]$

$y_1^{(1)} = 0.9133 + 0.05\left[-0.7341 + 0.2 - (0.8399)^2\right]$

$y_1^{(1)} = 0.9133 + 0.05\left[-0.7341 + 0.2 - 0.7054\right]$

$\boxed{y_1^{(1)} = 0.8513}$

$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$

$= 0.9133 + 0.05\left[-0.7341 + x_1 - (y_1^{(1)})^2\right]$

$= 0.9133 + 0.05\left[-0.7341 + 0.2 - (0.8513)^2\right]$

$= 0.9133 + 0.05$

$\boxed{y_1^{(2)} = 0.8504}$ Similarly $\boxed{y_1^{(3)} = 0.8504}$

Thus $\underline{y(0.2) = 0.8504}$

4) Using modified Euler's method find $y(20.2)$ and $y(20.4)$ given that $\frac{dy}{dx} = \log_{10}(x/y)$ with $y(20)=5$ taking $h=0.2$

>> We shall first compute $y(20.2)$ and use this value to compute $y(20.4)$

I Stage: By data $x_0 = 20$, $y_0=5$ and $h=0.2$

$$f(x,y) = \log_{10}(x/y)$$

$$\therefore f(x_0,y_0) = \log_{10}\left(\frac{x_0}{y_0}\right) = \log_{10}\left(\frac{20}{5}\right) = \log_{10}(4)$$

$$\boxed{f(x_0,y_0) = 0.6021}$$

$$x_1 = x_0+h = 20+0.2 = 20.2$$

$$\boxed{x_1 = 20.2}$$

$$y(x_1) = y(20.2) = ?$$

from Euler's formula

$$y_1^{(0)} = y_0 + hf(x_0,y_0)$$

$$y_1^{(0)} = 5+0.2\,(0.6021)$$

$$\boxed{y_1^{(0)} = 5.1204}$$

Now Euler's modified formula.

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1,y_1^{(0)})\right]$$

$$= 5+\frac{0.2}{2}\left[0.6021 + \log_{10}\left(\frac{x_1}{y_1^{(0)}}\right)\right]$$

$$= 5+0.1\left[0.6021+\log_{10}\left[\frac{20.2}{5.1204}\right]\right]$$

$$= 5+0.1\left[0.6021+\log_{10}[3.945]\right]$$

$$\boxed{y_1^{(1)} = 5.1198}$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 5 + 0.1\left[0.6021 + \log_{10}\left[\frac{x_1}{y_1^{(1)}}\right]\right]$$

$$= 5 + 0.1\left[0.6021 + \log_{10}\left[\frac{20.2}{5.1198}\right]\right]$$

$$= 5 + 0.1\left[0.6021 + \log_{10}(3.9455)\right]$$

$$\boxed{y_1^{(2)} = 5.1198}$$

Thus $\boxed{y(20.2) = 5.1198}$.

II Stage

Now, let $x_0 = 20.2$; $y_0 = 5.1198$

$$f(x, y) = \log_{10}\left(\frac{x}{y}\right) = \log_{10}\left(\frac{20.2}{5.1198}\right)$$

$$f(x_0, y_0) = \log_{10}(3.9455) = 0.5961$$

$$\boxed{f(x_0, y_0) = 0.5961}$$

$$x_1 = x_0 + h = 0.2 + 20.2 = 20.4$$

$\boxed{x_1 = 20.4}$. $\quad y(x_1) = y(20.4) = ?$ $\quad y(x_1) = y_1$

Euler's formula

$$y_1^{(0)} = y_0 + hf(x_0, y_0)$$

$$= 5.1198 + (0.2)(0.5961)$$

$$\boxed{y_1^{(0)} = 5.239}$$

Euler's modified formula

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 5.1198 + \frac{0.2}{2}\left[0.5961 + \log_{10}\left[\frac{x_1}{y_1^{(0)}}\right]\right]$$

$$= 5.1198 + 0.1\left[0.5961 + \log_{10}\left(\frac{20.4}{5.239}\right)\right]$$

$$= 5.1198 + 0.1\left[0.5961 + \log_{10}(3.8939)\right]$$

$$\boxed{y_1^{(1)} = 5.2384}$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 5.1198 + 0.1\left[0.5961 + \log_{10}\left[\frac{x_1}{y_1^{(1)}}\right]\right]$$

$$= 5.1198 + 0.1\left[0.5961 + \log_{10}\left(\frac{20.4}{5.2384}\right)\right]$$

$$= 5.1198 + 0.1\left[0.5961 + \log_{10}(3.8943)\right]$$

$$\boxed{y_1^{(2)} = 5.2385}$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 5.1198 + 0.1\left[0.5961 + \log_{10}\left(\frac{x_1}{y_1^{(2)}}\right)\right]$$

$$= 5.1198 + 0.1\left[0.5961 + \log_{10}\left[\frac{20.4}{5.2385}\right]\right]$$

$$= 5.1198 + 0.1\left[0.5961 + \log_{10}(3.8942)\right]$$

$$\boxed{y_1^{(3)} = 5.2385}$$

Thus *** $\boxed{y(20.4) = 5.2385}$ ***

5) Use modified Euler's method to solve $\frac{dy}{dx} = x + |\sqrt{y}|$ in the range $0 \le x \le 0.4$ by taking $h = 0.2$ given that $y=1$ at $x=0$ initially. (22) 8

>> we need to compute $y(0.2)$ and $y(0.4)$ with $h=0.2$.

I stage By data $x_0 = 0$, $y_0 = 1$, $f(x,y) = x + \sqrt{y}$

where the modulus sign indicates that the we have to take only the positive value of $\sqrt{y}$

$$f(x_0, y_0) = x_0 + \sqrt{y_0} = 0 + \sqrt{1} = 1$$

$$\therefore \boxed{f(x_0, y_0) = 1}$$

$$x_1 = x_0 + h = 0 + 0.2 = 0.2$$

$$\boxed{x_1 = 0.2} \qquad y(x_1) = y_1 \Rightarrow y(0.2) = ?$$

from Euler's formula

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$

$$= 1 + 0.2(1)$$

$$\boxed{y_1^{(0)} = 1.2}$$

we have modified Euler's formula

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 1 + \frac{0.2}{2}\left[1 + x_1 + \sqrt{y_1^{(0)}}\right]$$

$$= 1 + 0.1\left[1 + 0.2 + \sqrt{1.2}\right]$$

$$= 1 + 0.1\left[1.2 + 1.0954\right]$$

$$\boxed{y_1^{(1)} = 1.2295}$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1 + 0.1\left[1 + x_1 + \sqrt{y_1^{(1)}}\right]$$

$$= 1 + 0.1\left[1 + 0.2 + \sqrt{1.2295}\right]$$

$$= 1 + 0.1\left[1.2 + 1.1088\right]$$

$$\boxed{y_1^{(2)} = 1.2309}$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1 + 0.1\left[1 + x_1 + \sqrt{y_1^{(2)}}\right]$$

$$= 1 + 0.1\left[1 + 0.2 + \sqrt{1.2309}\right]$$

$$= 1 + 0.1\left[1.2 + 1.1095\right]$$

$$\boxed{y_1^{(3)} = 1.2310}$$

$$y_1^{(4)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(3)})\right]$$

$$= 1 + 0.1\left[1 + x_1 + \sqrt{y_1^{(3)}}\right]$$

$$= 1 + 0.1\left[1 + 0.2 + \sqrt{1.231}\right]$$

$$= 1 + 0.1\left[1.2 + 1.1095\right]$$

$$\boxed{y_1^{(4)} = 1.231}$$

Thus $\boxed{y(0.2) = 1.231}$

II Stage Now let : $x_0 = 0.2$, $y_0 = 1.231$

$f(x,y) = x + \sqrt{y}$

$f(x_0, y_0) = x_0 + \sqrt{y_0} = (0.2) + \sqrt{1.231} = 1.3095$

$\therefore \boxed{f(x_0, y_0) = 1.3095}$

$x_1 = x_0 + h = 0.2 + 0.2 = 0.4$

$\therefore \boxed{x_1 = 0.4}$ $y(x_1) = y_1 \Rightarrow y(0.4) = ?$

Sub Euler's formula,

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$= 1.231 + 0.2 (1.3095)$

$\boxed{y_1^{(0)} = 1.4929}$

Next modified Euler's formula

$y_1^{(1)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1.231 + \frac{0.2}{2} [1.3095 + x_1 + \sqrt{y_1^{(0)}}]$

$= 1.231 + 0.1 [1.3095 + 0.4 + \sqrt{1.4929}]$

$= 1.231 + 0.1 [1.3095 + 0.4 + 1.2218]$

$= 1.231 + 0.1 [2.9313]$

$= 1.231 + 0.2931$

$\boxed{y_1^{(1)} = 1.5241}$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1.231 + 0.1\left[1.3095 + x_1 + \sqrt{y_1^{(1)}}\right]$$

$$= 1.231 + 0.1\left[1.3095 + 0.4 + \sqrt{1.5241}\right]$$

$$= 1.231 + 0.1\left[1.3095 + 0.4 + 1.2345\right]$$

$$\boxed{y_1^{(2)} = 1.5254}$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1.231 + 0.1\left[\cancel{1}.3095 + x_1 + \sqrt{y_1^{(2)}}\right]$$

$$= 1.231 + 0.1\left[1.3095 + 0.4 + \sqrt{1.5254}\right]$$

$$= 1.231 + 0.1\left[1.3095 + 0.4 + 1.2351\right]$$

$$= 1.231 + 0.1\left[1.3095 + 0.4 +$$

$$\boxed{y_1^{(3)} = 1.5255}$$

$$y_1^{(4)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(3)})\right]$$

$$= 1.231 + 0.1\left[1.3095 + 0.4 + \sqrt{1.5255}\right]$$

$$= 1.231 + 0.1\left[1.7095 + 1.2351\right]$$

$$\boxed{y_1^{(4)} = 1.5255}$$

Thus, $\boxed{y(0.4) = 1.5255}$

https://hemanthrajhemu.github.io

6) Use modified Euler's method to compute $y(0.1)$ given that $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ by taking $h = 0.05$ considering the accuracy upto two approximations on each step.

>> We need to compute $y(0.05)$ first and use this value to compute $y(0.1)$

I Stage By data $x_0 = 0$ and $y_0 = 1$

$$f(x, y) = x^2 + y$$

$$\therefore f(x_0, y_0) = (0)^2 + 1 \quad \cancel{= 0.01 + 1 = 1.01}$$

$$\therefore \boxed{f(x_0, y_0) = 1}$$

$$x_1 = x_0 + h = 0 + 0.05$$

$$\boxed{x_1 = 0.05} \qquad y(x_1) = y_1 \Rightarrow y(0.05) = ?$$

from Euler's formula

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$

$$= 1 + 0.05(1) = 1.05$$

$$\therefore \boxed{y_1^{(0)} = 1.05}$$

Next modified Euler's method

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 1 + \frac{0.05}{2}\left[1 + x_1^2 + y_1^{(0)}\right]$$

$$= 1 + 0.025\left[1 + (0.05)^2 + 1.05\right]$$

$$= 1 + 0.025\left[1 + 0.0025 + 1.05\right]$$

$$\boxed{y_1^{(1)} = 1.0513}$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1 + 0.025\left[1 + x_1^2 + y_1^{(1)}\right]$$

$$= 1 + 0.025\left[1 + (0.05)^2 + 1.0513\right]$$

$$= 1 + 0.025\left[1.0025 + 1.0513\right]$$

$$= 1 + 0.025\left[2.0538\right]$$

$$\boxed{y_1^{(2)} = 1.0513}$$

Thus $y(0.05) = 1.0513$

II Stage Let $x_0 = 0.05$, $y_0 = 1.0513$

$f(x, y) = x^2 + y.$

$\therefore f(x_0, y_0) = x_0^2 + y_0 = (0.05)^2 + 1.0513$

$$\boxed{f(x_0, y_0) = 1.0538}$$

$x_1 = x_0 + h = 0.05 + 0.05 = 0.1$

$\boxed{x_1 = 0.1}$ $\therefore y(x_1) = y_1 \Rightarrow y(0.1) = ?$

Euler's formula

$y_1^{(0)} = y_0 + h f(x_0, y_0) = 1.0513 + 0.05(1.0538)$

$$\boxed{y_1^{(0)} = 1.1040}$$

modified Euler's formula

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 1.0513 + \frac{0.05}{2}\left[1.0538 + x_1^2 + y_1^{(0)}\right]$$

$$= 1.0513 + 0.025\left[1.0538 + (0.1)^2 + 1.104\right]$$

$$= 1.0513 + 0.025\left[1.0638 + 1.104\right]$$

$$\boxed{y_1^{(1)} = 1.1055}$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1.0513 + 0.025\left[1.0538 + x_1^2 + y_1^{(1)}\right]$$

$$= 1.0513 + 0.025\left[1.0638 + 1.1055\right]$$

$$= 1.0513 + 0.025$$

$$\boxed{y_1^{(2)} = 1.1055}$$

Then $y(0.1) = 1.1055$

7) Using Euler's predictor and corrector formulae solve $\frac{dy}{dx} = x + y$ at $x = 0.2$ given that $y(0) = 1$

>> We need to compute $y(0.2)$ and since the step size is not specified we shall take it to be 0.2 itself.

Remark: if we had worked the problem in two stages (taking $h = 0.1$) we would have got more accurate answer. It may be noted that lesser is the step size greater is accuracy.

By data we've $x_0 = 0$, $y_0 = 1$

I stage

$f(x, y) = x + y$

$\therefore f(x_0, y_0) = x_0 + y_0 = 0 + 1 = 1$

$\therefore \boxed{f(x_0, y_0) = 1}$ and $\boxed{h = 0.1}$ assumed value

$x_1 = x_0 + h = 0 + 0.1$

$\boxed{x_1 = 0.1}$ $y(x_1) = y_1 \Rightarrow y(0.1) = ?$

Euler's formula (predictor formula)

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$= 1 + 0.1 (1)$

$\boxed{y_1^{(0)} = 1.1}$

Euler's modified formula (corrector formula)

$y_1^{(1)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1 + \frac{0.1}{2} [1 + x_1 + y_1^{(0)}]$

$= 1 + 0.05 [1 + 0.1 + 1.1]$

$= 1 + 0.05 [1.1 + 1.1]$

$\boxed{y_1^{(1)} = 1.11}$

$y_1^{(2)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$= 1 + 0.05 [1 + x_1 + y_1^{(1)}]$

$= 1 + 0.05 [1.1 + 1.11]$

$\boxed{y_1^{(2)} = 1.1105}$

$y_1^{(3)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(2)})]$

$= 1 + 0.05 [1.1 + 1.1105]$

$= 1 + 0.05$

$\boxed{y_1^{(3)} = 1.1105}$

By data we have $x_0 = 0$, $y_0 = 1$

$f(x, y) = x + y$ and $h = 0.2$ (assumed value)

$f(x_0, y_0) = 0 + 1 = 1$

$\boxed{f(x_0, y_0) = 1}$

$x_1 = x_0 + h = 0 + 0.2$

$\boxed{x_1 = 0.2}$ $\quad y(x_1) = y_1 \Rightarrow y(0.2) = ?$

we have Euler's formula (predictor formula)

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$= 1 + 0.2(1)$

$\boxed{y_1^{(0)} = 1.2}$

modified Euler's formula (corrector formula)

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$

$= 1 + \frac{0.2}{2}\left[1 + x_1 + y_1^{(0)}\right]$

$= 1 + 0.1\left[1 + 0.2 + 1.2\right]$

$\boxed{y_1^{(1)} = 1.24}$

$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$

$\boxed{y_1^{(2)} = 1.244}$

$\boxed{y_1^{(3)} = 1.244}$ $\quad$ Thus the required

$\boxed{y_1^{(4)} = 1.2444}$ $\quad \boxed{y(0.2) = 1.2444}$

8) using Euler's predictor and corrector formula compute $y(1.1)$ correct to five decimal places given that $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ and $y=1$ at $x=1$,

Ans By data $\frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x} = \frac{1-xy}{x^2}$

we've $f(x,y) = \frac{1-xy}{x^2}$ $\quad x_0 = 1,\ y_0 = 1$

let us take $\underline{h=0.1}$

$f(x_0,y_0) = \frac{1-x_0y_0}{x_0^2} = \frac{1-1(1)}{1} = 0$

$\boxed{f(x_0,y_0)=0}$ $\quad x_1 = x_0+h = 1+0.1 = 1.1$

$\boxed{x_1 = 1.1}$ $\quad y(x_1) = y_1 \Rightarrow \underline{y(1.1) = ?}$

Euler's formula $y_1^{(0)} = y_0 + hf(x_0,y_0)$

$= 1 + 0.1(0) = 1$

$\therefore \boxed{y_1^{(0)} = 1}$

modified formula

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1,y_1^{(0)})\right]$

$= 1 + \frac{0.1}{2}\left[0 + \frac{1 - x_1y_1^{(0)}}{x_1^2}\right]$

$= 1 + 0.05\left[\frac{1 - 1.1(1)}{(1.1)^2}\right]$

$= 1 + 0.05\left[\frac{1-1.1}{1.21}\right]$

$= 1 + 0.05\left[1 - \cancel{0.9090}\right]$

$= 1 + 0.05\left[\frac{-0.1}{1.21}\right]$

$\boxed{y_1^{(1)} = 0.995\cancel{87}} \rightarrow \underline{0.99587}$

https://hemanthrajhemu.github.io

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1 + 0.05\left[0 + \frac{1 - x_1 y_1^{(1)}}{x_1^2}\right]$$

$$= 1 + 0.05\left[\frac{1 - 1.1(0.9959)}{(1.1)^2}\right]$$

$$= 1 + 0.05\left[\frac{1 - 1.0955}{1.21}\right]$$

$$= 1 + 0.05\left[-\frac{0.0955}{1.21}\right]$$

$$\boxed{y_1^{(2)} = 0.99605}$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1 + 0.05\left[0 + \frac{1 - x_1 y_1^{(2)}}{x_1^2}\right]$$

$$= 1 + 0.05\left[\frac{1 - 1.1(0.99605)}{1.21}\right]$$

$$\boxed{y_1^{(3)} = 0.99605}$$

Thus $\boxed{y(1.1) = 0.99605}$

Correct to 5 decimal place

Runge - Kutta method of fourth order

- Consider the IVP $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$, we need to find $y(x_0+h)$ where h is the step size.

we have to first compute k_1, k_2, k_3, k_4 by the following formulae.

$$k_1 = h f(x_0, y_0)$$

$$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = h f(x_0 + h, y_0 + k_3)$$

The required

$$y(x_0+h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Problems

1) Given $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$ compute $y(0.2)$ by taking $h = 0.2$ using R-K-method of 4^{th} order.

>> By data

$$f(x, y) = 3x + \frac{y}{2}$$

$$x_0 = 0, \quad y_0 = 1, \quad h = 0.2$$

we shall compute 1st

k_1, k_2, k_3, k_4

$K_1 = h f(x_0, y_0)$

$K_1 = 0.2 f(0, 1) = 0.2 \left[3x_0 + y_0/2\right]$

$= 0.2 \left[3(0) + 1/2\right]$

$= 0.2 \left[0 + 0.5\right]$

$\boxed{K_1 = 0.1}$

$K_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= h f\left(0 + \frac{0.2}{2}, 1 + \frac{0.1}{2}\right)$

$= (0.2) f(0.1, 1 + 0.05)$

$= 0.2 f(0.1, 1.05)$

$= 0.2 \left[3 \times 0.1 + \frac{1.05}{2}\right]$

$= 0.2 \left[0.3 + 0.525\right]$

$\boxed{K_2 = 0.165}$

$K_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.165}{2}\right)$

$= 0.2 f(0.1, 1.0825)$

$= 0.2 \left[3 \times 0.1 + \frac{1.0825}{2}\right]$

$= 0.2 \left[0.84125\right]$

$\boxed{K_3 = 0.16825}$

$K_4 = h f(x_0 + h, y_0 + K_3)$

$K_4 = 0.2 f(0 + 0.2, 1 + 0.16825)$

$K_4 = 0.2 f(0.2, 1.16825)$

$K_4 = 0.2 \left[3 \times 0.2 + \frac{1.16825}{2}\right]$

$K_4 = 0.2 \left[1.184125\right]$

$\boxed{K_4 = 0.236825}$

Thus we've

$y(x_0 + h) = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$\therefore y(0.2) = y_0 + \frac{1}{6}(\ \text{''}\ )$

$\therefore y(0.2) = 1 + \frac{1}{6}\left(0.1 + 2(0.165) + 2(0.16825) + 0.236825\right)$

$\underline{y(0.2) = 1.1672}$

2) Use fourth order R-K-method to solve $(x+y)\frac{dy}{dx}=1$, $y(0.4)=1$ at $x=0.5$ correct to four decimal places.

>> we have $\frac{dy}{dx}=\frac{1}{x+y}$ and $y_0=1$ at $x_0=0.4$

$f(x,y)=\frac{1}{x+y}$, $x_0=0.4$, $y_0=1$ $y(0.5)=?$

Here $x_0+h=0.5$

$\therefore h=0.5-x_0=0.5-0.4=0.1 \Rightarrow \boxed{h=0.1}$

we shall first compute k_1, k_2, k_3, k_4

$k_1 = hf(x_0, y_0)$
$= 0.1\, f(0.4, 1)$
$= 0.1\left[\frac{1}{0.4+1}\right]$
$= 0.1\left[\frac{1}{1.4}\right]$
$= 0.1\,[0.7143]$
$\boxed{k_1 = 0.0714}$

$k_2 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right)$
$= 0.1\, f\left(0.4+\frac{0.1}{2}, 1+\frac{0.0714}{2}\right)$
$= 0.1\, f(0.4+0.05, 1+0.0357)$
$= 0.1\, f(0.45, 1.0357)$
$= 0.1\left[\frac{1}{0.45+1.0357}\right]$

$k_2 = 0.1\,[0.6731]$
$\boxed{k_2 = 0.0673}$

$$k_3 = h f\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2}\right)$$

$$= 0.1\ f\left(0.4 + \frac{0.1}{2},\ 1 + \frac{0.0673}{2}\right)$$

$$= 0.1\ f(0.45,\ 1.03365)$$

$$= 0.1\left[\frac{1}{0.45 + 1.03365}\right]$$

$$\boxed{k_3 = 0.0674}$$

$$k_4 = h f(x_0 + h,\ y_0 + k_3)$$

$$= 0.1\ f(0.4 + 0.1,\ 1 + 0.0674)$$

$$= 0.1\ f(0.5,\ 1.0674)$$

$$= 0.1\left[\frac{1}{0.5 + 1.0674}\right]$$

$$\boxed{k_4 = 0.0638}$$

we have

$$y(x_0 + h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$\therefore\ y(0.5) = 1 + \frac{1}{6}\left[0.0714 + 2(0.0673) + 2(0.0674) + 0.0638\right]$$

$$\boxed{y(0.5) = 1.0674}$$

3) Using Runge-kutta method of 4th order, find $y(0.2)$ for the eqn $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.2$

>> By data $f(x,y) = \frac{y-x}{y+x}$, $x_0 = 0$, $y_0 = 1$, $h = 0.2$

we shall first compute k_1, k_2, k_3, k_4

$k_1 = hf(x_0, y_0)$

$k_1 = 0.2 f(0, 1)$

$= 0.2 \left[\frac{1-0}{1+0}\right]$

$= 0.2 [1]$

$\boxed{k_1 = 0.2}$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right)$

$= 0.2 f(0.1, 1 + 0.1)$

$= 0.2 f(0.1, 1.1)$

$= 0.2 \left[\frac{1.1 - 0.1}{1.1 + 0.1}\right]$

$= 0.2 \left[\frac{1}{1.2}\right]$

$= 0.2 (0.8333)$

$\boxed{k_2 = 0.1667}$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.2 f\left(0.1, 1 + \frac{0.1667}{2}\right)$

$= 0.2 f(0.1, 1 + 0.08335)$

$= 0.2 f(0.1, 1.08335)$

$= 0.2 \left[\frac{1.08335 - 0.1}{1.08335 + 0.1}\right]$

$\boxed{k_3 = 0.1662}$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.2 f(0 + 0.2, 1 + 0.1662)$

$= 0.2 f(0.2, 1.1662)$

$= 0.2 \left[\frac{1.1662 - 0.2}{1.1662 + 0.2}\right]$

$= 0.2 [0.7072]$

$\boxed{k_4 = 0.1414}$

$$y(x_0+h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y(0.2) = 1 + \frac{1}{6}(0.2 + 2(0.1667) + 2(0.1662) + 0.1414)$$

$$\boxed{y(0.2) = 1.1679}$$

4) Use fourth order R-K. method to find y at $x = 0.1$ given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$ and $h = 0.1$

>> By data $f(x,y) = 3e^x + 2y$, $x_0 = 0$, $y_0 = 0$, $h = 0.1$

we shall first compute k_1, k_2, k_3, k_4

$$k_1 = hf(x_0, y_0)$$
$$= 0.1 f(0, 0)$$
$$= 0.1 [3e^0 + 2(0)]$$
$$= 0.1 [3 + 0]$$
$$\boxed{k_1 = 0.3}$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$
$$= 0.1 f\left(0 + \frac{0.1}{2}, 0 + \frac{0.3}{2}\right)$$
$$= 0.1 f(0.05, 0.15)$$
$$= 0.1 [3e^{0.05} + 2(0.15)]$$
$$= 0.1 [3.4538]$$
$$\boxed{k_2 = 0.3454}$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$
$$= 0.1 f\left(0.05, 0 + \frac{0.3454}{2}\right)$$
$$= 0.1 f(0.05, 0.1727)$$
$$= 0.1 [3e^{0.05} + 2(0.1727)]$$
$$\boxed{k_3 = 0.3499}$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$
$$= 0.1 f(0.1, 0.3499)$$
$$= 0.1 [3e^{0.1} + 2(0.3499)]$$
$$\boxed{k_4 = 0.4015}$$

we have

$$y(x_0+h) = y_0 + \frac{1}{6}(k_1+2k_2+2k_3+k_4)$$

$$y(0.1) = 0 + \frac{1}{6}(0.3+2(0.3454)+2(0.3499)+0.4015)$$

$$\boxed{y(0.1) = 0.3487}$$

5) Use fourth order R-K. method to compute $y(1.1)$ given that $\frac{dy}{dx} = xy^{1/3}$, $y(1)=1$

→ By data $f(x,y) = xy^{1/3}$, $x_0 = 1$, $y_0 = 1$

we need to compute $y(1.1)$

This implies that $x_0+h = 1.1$

$$\therefore h = 1.1 - 1 = 0.1$$

$$\boxed{h = 0.1}$$

we shall compute k_1, k_2, k_3, k_4

$$k_1 = hf(x_0, y_0)$$

$$k_1 = 0.1 f(1,1) = 0.1 [1 \; 1^{1/3}] = 0.1 ((1)^{1/3})$$

$$= 0.1(1) = 0.1$$

$$\boxed{k_1 = 0.1}$$

$$k_2 = hf(x_0+\tfrac{h}{2}, y_0+\tfrac{k_1}{2})$$

$$k_2 = 0.1 f(1+\tfrac{0.1}{2}, 1+\tfrac{0.1}{2})$$

$$= 0.1 f(1+0.05, 1+0.05)$$

$$= 0.1 f(1.05, 1.05)$$

$$= 0.1 [(1.05)\times(1.05)^{1/3}]$$

$$\boxed{k_2 = 0.1067}$$

$$k_3 = hf(x_0+\tfrac{h}{2}, y_0+\tfrac{k_2}{2})$$

$$= 0.1 f(1.05, 1+\tfrac{0.1067}{2})$$

$$= 0.1 f(1.05, 1.05335)$$

$$= 0.1 [(1.05)(1.05335)^{1/3}]$$

$$\boxed{k_3 = 0.1068}$$

$K_4 = hf(x_0+h, y_0+k_3)$

$= 0.1\, f(1+0.1, 1+0.1068)$

$= 0.1\, f(1.1, 1.1068)$

$= 0.1\,(1.1 \times (1.1068)^{1/3})$

$\boxed{K_4 = 0.1138}$

we've

$y(x_0+h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 1 + \frac{1}{6}(0.1 + 2(0.1067) + 2(0.1068) + 0.1138)$

$\boxed{y(1.1) = 1.1068}$

6) Using R.K. method of 4th order $\frac{dy}{dx} + y = 2x$ at $x = 1.1$ given that $y = 3$ at $x = 1$ initially.

>> we have

$\frac{dy}{dx} = 2x - y$, $x_0 = 1$, $y_0 = 3$

$f(x,y) = 2x - y$

$x_0 + h = 1.1$

$\therefore h = 1.1 - x_0$

$h = 1.1 - 1 = 0.1$

$\boxed{h = 0.1}$

$k_1 = hf(x_0, y_0)$

$k_1 = hf(1, 3)$

$k_1 = h\,[2(1) - 3]$

$= 0.1\,[2 - 3]$

$= 0.1(-1)$

$\boxed{k_1 = -0.1}$

$$k_2 = hf\left(x_0+\frac{h}{2},\ y_0+\frac{k_1}{2}\right)$$

$$= 0.1\ f\left(1+\frac{0.1}{2},\ 3+\frac{(-0.1)}{2}\right)$$

$$= 0.1\ f\left(1+0.05,\ 3-0.05\right)$$

$$= 0.1\ f(1.05,\ 2.95)$$

$$= 0.1\ [2(1.05)-2.95]$$

$$\boxed{k_2 = -0.085}$$

$$k_3 = hf\left(x_0+\frac{h}{2},\ y_0+\frac{k_2}{2}\right)$$

$$= 0.1 f\left(1.05,\ 3+\left(\frac{-0.085}{2}\right)\right)$$

$$= 0.1 f(1.05,\ 3-0.0425)$$

$$= 0.1 f(1.05,\ 2.9575)$$

$$= 0.1\ [2(1.05)-2.9575]$$

$$\boxed{k_3 = -0.0858}$$

$$k_4 = hf(x_0+h,\ y_0+k_3)$$

$$= 0.1\ f(1+0.1,\ 3+(-0.0858))$$

$$= 0.1\ f(1.1,\ 2.9142)$$

$$= 0.1\ [2(1.1)-2.9142]$$

$$\boxed{k_4 = -0.0714}$$

Then $y(x_0+h) = y_0 + \frac{1}{6}(k_1+2k_2+2k_3+k_4)$

$$y(1.1) = 3+\frac{1}{6}[(-0.1)+2(-0.085)+2(-0.0858) + (-0.0714)]$$

$$\boxed{y(1.1) = 2.9145}$$

7) Using R-K. method of 4th order find $y(0.2)$ for the eqn $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0)=1$ taking $h=0.1$

I stage: $f(x,y) = \frac{y-x}{y+x}$, $x_0=0$, $y_0=1$, $h=0.1$

we shall first compute k_1, k_2, k_3, k_4

$k_1 = h f(x_0, y_0)$
$= 0.1 f(0,1)$
$= 0.1 \left[\frac{1-0}{1+0}\right]$
$= 0.1(1)$

$\boxed{k_1 = 0.1}$

$k_2 = h f\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right)$
$= 0.1 f\left(0+\frac{0.1}{2}, 1+\frac{0.1}{2}\right)$
$= 0.1 f(0.05, 1+0.05)$
$= 0.1 f(0.05, 1.05)$
$= 0.1 \left[\frac{1.05-0.05}{1.05+0.05}\right]$
$= 0.1(0.9091)$
$= 0.09091$

$\boxed{k_2 = 0.0909}$

$k_3 = h f\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right)$
$= 0.1 f\left(0.05, 1+\frac{0.0909}{2}\right)$
$= 0.1 f(0.05, 1.04545)$
$= 0.1 \left[\frac{1.04545-0.05}{1.04545+0.05}\right]$

$\boxed{k_3 = 0.0909}$

$k_4 = h f(x_0+h, y_0+k_3)$
$= 0.1 f(0+0.1, 1+0.0909)$
$= 0.1 f(0.1, 1.0909)$
$= 0.1 \left[\frac{1.0909-0.1}{1.0909+0.1}\right]$

$\boxed{k_4 = 0.0832}$

we have

$$y(x_0+h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y(0.1) = 1 + \frac{1}{6}[0.1 + 2(0.0909) + 2(0.0909) + 0.0832]$$

$$\boxed{y(0.1) = 1.0911}$$

II Stage $f(x,y) = \frac{y-x}{y+x}$; $x_0 = 0.1$, $y_0 = 1.0911$, $h = 0.1$

Calculate k_1, k_2, k_3, k_4

$$k_1 = hf(x_0, y_0)$$

$$k_1 = 0.1\, f(0.1, 1.0911)$$

$$= 0.1\left[\frac{1.0911 - 0.1}{1.0911 + 0.1}\right]$$

$$\boxed{k_1 = 0.0832}$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= 0.1\, f\left(0.1 + \frac{0.1}{2}, 1.0911 + \frac{0.0832}{2}\right)$$

$$= 0.1\, f(0.1 + 0.05, 1.0911 + 0.0416)$$

$$= 0.1\, f(0.15, 1.1327)$$

$$= 0.1\left[\frac{1.1327 - 0.15}{1.1327 + 0.15}\right]$$

$$\boxed{k_2 = 0.0766}$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= 0.1\, f\left(0.1 + \frac{0.1}{2}, 1.0911 + \frac{0.0766}{2}\right)$$

$$= 0.1\, f(0.15, 1.1294)$$

$$= 0.1\left[\frac{1.1294 - 0.15}{1.1294 + 0.15}\right]$$

$$\boxed{k_3 = 0.0766}$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$= 0.1\, f(0.1 + 0.1, 1.0911 + 0.0766)$$

$$= 0.1\, f(0.2, 1.1677)$$

$$= 0.1\left[\frac{1.1677 - 0.2}{1.1677 + 0.2}\right]$$

$$\boxed{k_4 = 0.0708}$$

Now $y(x_0+h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$$y(0.1+0.1) = 1.0912 + \frac{1}{6}(0.0832 + 2(0.0766) + 2(0.0766) + 0.0708)$$

$$\boxed{y(0.2) = 1.1678}$$

8) Solve : $(y^2-x^2)\,dx = (y^2+x^2)\,dy$ for $x = 0(0.2)0.4$ given that $y=1$ at $x=0$ initially, by applying R-K-method of order 4.

>> we have $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$; $x_0 = 0$, $y_0 = 1$, $h = 0.2$

<u>I Stage</u> $f(x,y) = \frac{y^2-x^2}{y^2+x^2}$

$k_1 = hf(x_0, y_0)$

$= 0.2\,f(0,1)$

$= 0.2\left[\frac{1^2-0^2}{1^2+0^2}\right]$

$$\boxed{k_1 = 0.2}$$

$k_2 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right)$

$= 0.2\,f\left(0+\frac{0.2}{2}, 1+\frac{0.2}{2}\right)$

$= 0.2\,f(0.1, 1+0.1)$

$= 0.2\,f(0.1, 1.1)$

$= 0.2\left[\frac{(1.1)^2-(0.1)^2}{(1.1)^2+(0.1)^2}\right]$

$$\boxed{k_2 = 0.1967}$$

$$k_3 = hf\left(x_0+\frac{h}{2},\ y_0+\frac{k_2}{2}\right)$$

$$= 0.2\, f\left(0.1,\ 1+\frac{0.1967}{2}\right)$$

$$= 0.2\, f(0.1,\ 1.09835)$$

$$= 0.2\left[\frac{(1.09835)^2-(0.1)^2}{(1.09835)^2+(0.1)^2}\right]$$

$$\boxed{k_3 = 0.1967}$$

$$k_4 = hf(x_0+h,\ y_0+k_3)$$

$$= 0.2\, f(0+0.2,\ 1+0.1967)$$

$$= 0.2\, f(0.2,\ 1.1967)$$

$$= 0.2\left[\frac{(1.1967)^2-(0.2)^2}{(1.1967)^2+(0.2)^2}\right]$$

$$\boxed{k_4 = 0.1891}$$

Thus $y(x_0+h) = y_0 + \frac{1}{6}(k_1+2k_2+2k_3+k_4)$

$$y(0+0.2) = 1+\frac{1}{6}\left[0.2+2(0.1967)+2(0.1967)+0.1891\right]$$

$$y(0.2) = 1.195983333$$

$$** \boxed{y(0.2) = 1.1960} **$$

II Stage

$f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$, $x_0 = 0.2$, $y_0 = 1.196$, $h = 0.2$

$$k_1 = h f(x_0, y_0) = 0.2 f(0.2, 1.196) = 0.2\left[\frac{(1.196)^2 - (0.2)^2}{(1.196)^2 + (0.2)^2}\right]$$

$$\boxed{k_1 = 0.1891}$$

$$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= 0.2 f\left(0.2 + \frac{0.2}{2}, 1.196 + \frac{0.1891}{2}\right)$$

$$= 0.2 f(0.2 + 0.1, 1.196 + 0.09455)$$

$$= 0.2 f(0.3, 1.29055)$$

$$= 0.2\left[\frac{(1.29055)^2 + (0.3)^2}{(1.29055)^2 + (0.3)^2}\right]$$

$$\boxed{k_2 = 0.1795}$$

$$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= 0.2 f\left(0.3, 1.196 + \frac{0.1795}{2}\right)$$

$$= 0.2 f(0.3, 1.28575)$$

$$= 0.2\left[\frac{(1.28575)^2 - (0.3)^2}{(1.28575)^2 + (0.3)^2}\right]$$

$$\boxed{k_3 = 0.1793}$$

$$k_4 = h f(x_0 + h, y_0 + k_3)$$

$$= 0.2 f(0.2 + 0.2, 1.196 + 0.1793)$$

$$= 0.2 f(0.4, 1.3753)$$

$$= 0.2\left[\frac{(1.3753)^2 - (0.4)^2}{(1.3753)^2 + (0.4)^2}\right]$$

$$\boxed{k_4 = 0.1688}$$

Thus $y(x_0 + h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$$y(0.4) = 1.196 + \frac{1}{6}(0.1891 + 2(0.1795) + 2(0.1793) + 0.1795)$$

$$\boxed{y(0.4) = 1.377}$$

Predictor and corrector metheods

Here we discuss two predictor and corrector metheods namely.

1) Milne's metheod

2) Adams - Bashforth metheod

Consider the d.eqn $y' = \frac{dy}{dx} = f(x, y)$ with a set of four predetermined values of y:

$y(x_0) = y_0$, $y(x_1) = x_1$, $y(x_2) = x_2$, $y(x_3) = y_3$

Here x_0, x_1, x_2, x_3 are equally spaced values of x with width h.

Also $x_4 = x_3 + h = x_0 + 4h$

predictor and corrector formulae to compute $y(x_4) = y_4$ are as follows,

Milne's predictor and corrector formulae

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3') \quad \text{--- (predictor formula)}$$

$$y_4^{(C)} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4') \quad \text{--- (corrector formula)}$$

Note:- These two formulae can be written in the general form as follows

$$y_{n+1}^{(P)} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$$y_{n+1}^{(c)} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

Adams - Bashforth predictor and corrector formulae

$$y_4^{(P)} = y_3 + \frac{h}{24}\left(55y_3' - 59y_2' + 37y_1' - 9y_0'\right)$$ predictor formula

$$y_4^{(c)} = y_3 + \frac{h}{24}\left(9y_4' + 19y_3' - 5y_2' + y_1'\right)$$ Corrector formula

Note: we can write down the general form of these two formulae also as in milne's method.

1). Given that $\frac{dy}{dx} = x - y^2$ and the data

$y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$

Compute y at $x = 0.8$ by applying

a) Milne's method b) Adams - Bashforth method

>> we prepare the following table using the given data which is essentially required for applying the predictor and corrector formulae.

x	y	$y' = x - y^2$
$x_0 = 0$	$y_0 = 0$	$y_0' = 0 - 0^2 = 0$
$x_1 = 0.2$	$y_1 = 0.02$	$y_1' = 0.2 - (0.02)^2 = 0.1996$
$x_2 = 0.4$	$y_2 = 0.0795$	$y_2' = 0.4 - (0.0795)^2 = 0.3937$
$x_3 = 0.6$	$y_3 = 0.1762$	$y_3' = 0.6 - (0.1762)^2 = 0.5690$
$x_4 = 0.8$	$y_4 = ?$	

a) By milne's method

wkt $x_4 = x_3 + h$

$h = x_4 - x_3 = 0.8 - 0.6$

$\boxed{h = 0.2}$

we've the predictor formula

$$y_4^{(P)} = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$\therefore y_4^{(P)} = 0 + \frac{4(0.2)}{3}\left[2(0.1996) - 0.3937 + 2(0.5690)\right]$$

$$\boxed{y_4^{(P)} = 0.3049}$$

Now $y_4' = f(x_4, y_4) = x_4 - y_4^2 = (0.8) - (0.3049)^2$

$$\boxed{y_4' = 0.7070}$$

Next corrector formula

$$y_4^{(c)} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$\therefore y_4^{(c)} = 0.0795 + \frac{0.2}{3}\left[0.3937 + 4(0.5689) + 0.7070\right]$$

$$\boxed{y_4^{(c)} = 0.3046}$$

Now $y_4' = x_4 - y_4^2$

$$= (0.8) - (0.3046)^2$$

$$\boxed{y_4' = 0.7072}$$

Subtituting this value of y_4' again in the corrector formula.

$$y_4^{(c)} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 0.0795 + \frac{0.2}{3}\left[0.3937 + 4(0.5690) + 0.7072\right]$$

$$\boxed{y_4^{(c)} = 0.3046}$$

Thus

** $\boxed{y_4 = y(0.8) = 0.3046}$ **

b) By Adams - Bashforth method

we've predictor formula

$$y_4^{(P)} = y_3 + \frac{h}{24}(55y_3' - 59y_2' + 37y_1' - 9y_0')$$

$$\therefore y_4^{(P)} = 0.1762 + \frac{0.2}{24}\left[55(0.5690) - 59(0.3937) + 37(0.1996) - 9(0)\right]$$

$$\boxed{y_4^{(P)} = 0.3050}$$

Now

$$y_4' = f(x_4, y_4)$$
$$= x_4 - y_4^2$$
$$= (0.8) - (0.3050)^2$$
$$\boxed{y_4' = 0.7070}$$

corrector formula

$$y_4^{(C)} = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1') \longrightarrow (1)$$

$$= 0.1762 + \frac{0.2}{24}(9(0.7070) + 19(0.5690) - 5(0.3937) + 0.1996)$$

$$\boxed{y_4^{(C)} = 0.3046}$$

Now $y_4' = f(x_4, y_4) = x_4 - y_4^2 = 0.8 - (0.3046)^2$

$$\boxed{y_4' = 0.7072}$$

Applying the corrector formulae again with only change on the value of y_4' we obtain.

$$y_4^{(C)} = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1')$$

$$(1) \Rightarrow \quad y_4^{(C)} = \text{———} (\;\; " \;\; \text{———} + \cancel{0.7072})$$

$$y_4^{(1)} = 0.1762 + \frac{0.2}{24}(9(0.7072) + \;\; " \;\;)$$

$$\boxed{y_4^{(C)} = 0.3046}$$

2) Apply Milne's method to compute $y(1.4)$ correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and following the data

$y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$

>> first we shall prepare the foll^ng table

x	y	$y' = x^2 + y/2$
$x_0 = 1$	$y_0 = 2$	$y_0' = 1^2 + \frac{2}{2} = 1+1 = 2$
$x_1 = 1.1$	$y_1 = 2.2156$	$y_1' = (1.1)^2 + \frac{2.2156}{2} = 2.3178$
$x_2 = 1.2$	$y_2 = 2.4649$	$y_2' = (1.2)^2 + \frac{2.4649}{2} = 2.6725$
$x_3 = 1.3$	$y_3 = 2.7514$	$y_3' = (1.3)^2 + \frac{2.7514}{2} = 3.0657$
$x_4 = 1.4$	$y_4 = ?$	

$x_4 = x_3 + h$

$h = x_4 - x_3$

$h = 1.4 - 1.3$

$\boxed{h = 0.1}$

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$

$$y_4^{(P)} = 2 + \frac{4 \times 0.1}{3}(2(2.3178) - 2.6725 + 2(3.0657))$$

$$\boxed{y_4^{(P)} = 3.0793}$$

Hence $y_4'^{P} = f(x_4, y_4)$

$$y_4'^{P} = x_4^2 + \frac{y_4}{2}$$

$$= (1.4)^2 + \frac{3.0793}{2}$$

$$= 3.49965$$

$$\boxed{y_4'^{P} = 3.4997}$$

Now consider

$$y_4^{(c)} = y_2 + \frac{h}{3}\left(y_2' + 4y_3' + y_4'\right) \quad \text{—— (1)}$$

$$= 2.4649 + \frac{0.1}{3}\left[2.6725 + 4(3.0657) + 3.4997\right]$$

$$\boxed{y_4^{(c)} = 3.0794}$$

Now $y_4^{(c)} \Rightarrow y_4' = f(x_4, y_4)$.

$$\therefore \; y_4' = x_4^2 + \frac{y_4}{2}$$

$$y_4' = (1.4)^2 + \frac{3.0794}{2}$$

$$\boxed{y_4' = 3.4997}$$

Substituting this value in y_4' again in the corrector formula

(1) $\Rightarrow$

$$y_4^{(c)} = y_2 + \frac{h}{3}\left(y_2' + 4y_3' + y_4'\right)$$

$$y_4^{(c)} = 2.4649 + \frac{0.1}{3}\left(2.6725 + 4(3.0657) + 3.4997\right)$$

$$y_4^{(c)} = 2.4649 + 0.92175$$

$$*** \quad \boxed{y_4^{(c)} = 3.0794} \quad ***$$

3) Use Taylor's series method (upto 3rd derivative term) to find y at $x = 0.1, 0.2, 0.3$ given that $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 1$

Apply milne's predictor - corrector formula to find $y(0.4)$ using the generated set of initial values.

>> By Taylor's series one of the problem

$y(0.1) = 1.1113$, $y(0.2) = 1.2507$, $y(0.3) = 1.426$

with $y(0) = 1$. we prepare the table,

x	y	$y' = x^2 + y^2$
$x_0 = 0$	$y_0 = 1$	$y_0' = 0^2 + 1^2 = 1$
$x_1 = 0.1$	$y_1 = 1.1113$	$y_1' = (0.1)^2 + (1.1113)^2 = 1.245$
$x_2 = 0.2$	$y_2 = 1.2507$	$y_2' = (0.2)^2 + (1.2507)^2 = 1.6043$
$x_3 = 0.3$	$y_3 = 1.426$	$y_3' = (0.3)^2 + (1.426)^2 = 2.1235$
$x_4 = 0.4$	$y_4 = ?$	

Consider $y_4^{(P)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$

$\therefore y_4^{(P)} = 1 + \frac{4(0.1)}{3}\left[2(1.245) - 1.6043 + 2(2.1235)\right]$

$\boxed{y_4^{(P)} = 1.6844}$

$x_4 = x_3 + h$

$x_4 - x_3 = h$

$0.4 - 0.3 = h$

$\boxed{h = 0.1}$

Now
$$y_4' = f(x_4, y_4)$$
$$= x_4^2 + y_4^2$$
$$= (0.4)^2 + (1.6844)^2$$
$$\boxed{y_4' = 2.9972}$$

Next $$y_4^{(c)} = y_2 + \frac{h}{3}\left(y_2' + 4y_3' + y_4'\right) \quad \text{— (1)}$$

$$y_4^{(c)} = 1.2507 + \frac{0.1}{3}\left(1.6043 + 4(2.1235) + 2.9972\right)$$
$$\boxed{y_4^{(c)} = 1.6872}$$

Now $$y_4' = f(x_4, y_4)$$
$$= x_4^2 + y_4^2$$
$$= (0.4)^2 + (1.6872)^2$$
$$\boxed{y_4' = 3.0066}$$

(1) $\Rightarrow$
$$y_4^{(c)} = 1.2507 + \frac{0.1}{3}\left[1.6043 + 4(2.1235) + 3.0066\right]$$
$$= 1.2507 + 0.043683$$
$$y_4^{(c)} = 1.68753 \simeq 1.6875$$

That $$\boxed{y(0.4) = 1.6875}$$

4) The foll^{ng} table gives the sol^{n} of $5xy' + y^2 - 2 = 0$. Find the value of y at $x = 4.5$ using milne's predictor and corrector formulae. use the corrector formula twice.

x	4	4.1	4.2	4.3	4.4
y	1	1.0049	1.0097	1.0143	1.0187

>> By data
$5xy' + y^2 - 2 = 0$ (or) $5xy' = 2 - y^2$

$y' = \frac{2 - y^2}{5x}$

we prepare the table

x	y	$y' = \frac{2-y^2}{5x}$
$x_0 = 4$	$y_0 = 1$	$y_0' = \frac{2 - 1^2}{5(4)} = 0.05$
$x_1 = 4.1$	$y_1 = 1.0049$	$y_1' = \frac{2 - (1.0049)^2}{5(4.1)} = 0.0483$
$x_2 = 4.2$	$y_2 = 1.0097$	$y_2' = \frac{2 - (1.0097)^2}{5(4.2)} = 0.0467$
$x_3 = 4.3$	$y_3 = 1.0143$	$y_3' = \frac{2 - (1.0143)^2}{5(4.3)} = 0.0452$
$x_4 = 4.4$	$y_4 = 1.0187$	$y_4' = \frac{2 - (1.0187)^2}{5(4.4)} = 0.0437$
$x_5 = 4.5$	$y_5 = ?$	

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$

$$y_4^{(c)} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4')$$

Since we require y_5, the equivalent form of these formulae is given by

$$y_5^{(P)} = y_1 + \frac{4h}{3}(2y_2' - y_3' + 2y_4') \longleftarrow ①$$

$$y_5^{(c)} = y_3 + \frac{h}{3}(y_3' + 4y_4' + y_5') \longleftarrow ②$$

$x_4 = x_3 + h$
$x_5 = x_4 + h$
$h = x_5 - x_4$
$h = 4.5 - 4.4$
$\boxed{h = 0.1}$

Hence ① $\Rightarrow$

$$y_5^{(P)} = 1.0049 + \frac{4(0.1)}{3}\left[2(0.0467) - 0.0452 + 2(0.0437)\right]$$

$$= 1.02298 \simeq 1.023$$

$$\boxed{y_5^{(P)} = 1.023}$$

Now $y_5' = f(x_5, y_5)$

$$= \frac{2 - y_5^2}{5x_4}$$

$$= \frac{2 - (1.023)^2}{5(4.5)}$$

$$\boxed{y_5' = 0.0424}$$

② ⇒

$$y_5^{(c)} = 1.0143 + \frac{0.1}{3}\left(0.0452 + 4(0.0437) + 0.0424\right)$$

$$\boxed{y_5^{(c)} = 1.023}$$

Thus $y(4.5) = 1.023$

5) if $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$ and $y(0.3) = 2.090$, find $y(0.4)$ correct to four decimal places by using

a) milne's predictor - corrector formula

b) Adams - Bashforth predictor - corrector formula (Apply corrector formula twice)

x	y	$y' = 2e^x - y$
$x_0 = 0$	$y_0 = 2$	$y_0' = 2e^{(0)} - 2 = 0$
$x_1 = 0.1$	$y_1 = 2.010$	$y_1' = 2e^{(0.1)} - 2.010 = 0.2003$
$x_2 = 0.2$	$y_2 = 2.040$	$y_2' = 2e^{(0.2)} - 2.040 = 0.4028$
$x_3 = 0.3$	$y_3 = 2.090$	$y_3' = 2e^{(0.3)} - 2.090 = 0.6097$
$x_4 = 0.4$	$y_4 = ?$	

$x_4 = x_3 + h$

$h = x_4 - x_3 = 0.4 - 0.3$

$$\boxed{h = 0.1}$$

a) Milne's P-C. method

- $y_4^{(P)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$

$\therefore y_4^{(P)} = 2 + \frac{4\times0.1}{3}\left(2(0.2003) - 0.4028 + 2(0.6097)\right]$

$\boxed{y_4^{(P)} = 2.1623}$

Now $y_4' = 2e^{0.4} - 2.1623 = 0.8213$

$\boxed{y_4' = 0.8213}$

$y_4^{(C)} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4')$ —— ①

$\therefore y_4^{(C)} = 2.04 + \frac{0.1}{3}\left(0.4028 + 4(0.6097) + 0.8213\right)$

$\boxed{y_4^{(C)} = 2.1621}$

Now $y_4' = 2e^{0.4} - 2.1621$

$\boxed{y_4' = 0.8215}$

Now ① ⇒

$y_4^{(C)} = 2.04 + \frac{0.1}{3}\left(0.4028 + 4(0.6097) + 0.8215\right)$

$y_4^{(C)} = 2.162103333 \simeq 2.1621$

Thus $\boxed{y_4^{(C)} = 2.1621}$

** $\boxed{y(0.4) = 2.1621}$ **

b) Adams - Bashforth P-C - method.

$$y_4^{(P)} = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right)$$

$$y_4^{(P)} = 2.09 + \frac{0.1}{24}\left[55(0.6097) - 59(0.4028) + 37(0.2003) - 9(0)\right]$$

$$\boxed{y_4^{(P)} = 2.1616}$$

Now $y_4' = f(x_4, y_4)$

$$y_4' = 2e^x - y$$

$$y_4' = 2e^{0.4} - 2.1616$$

$$\boxed{y_4' = 0.822}$$

$$y_4^{(c)} = y_3 + \frac{h}{24}\left(9y_4' + 19y_3' - 5y_2' + y_1'\right) \quad \text{--- (1)}$$

$$\therefore y_4^{(c)} = 2.09 + \frac{0.1}{24}\left[9(0.822) + 19(0.6097) - 5(0.4028) + 0.2003\right]$$

$$\boxed{y_4^{(c)} = 2.1615}$$

Now $y_4' = 2e^{0.4} - 2.1615 = 0.8221\cancel{5}9395$

$$\boxed{y_4' = 0.8221}$$

(1) $\Rightarrow$ $y_4^{(c)} = 2.1615$

Thus $\boxed{y(0.4) = 2.1615}$

6) Apply Adams-Bashforth method to solve (42)
the eqn $(y^2+1)dy - x^2 dx = 0$ at $x=1$, given $y(0)=1$
by generating the initial values from picard's
2nd approximation at a step size of 0.25.
Apply the corrector formula twice.

>> By data

$$\frac{dy}{dx} = y' = \frac{x^2}{y^2+1}$$

Refer one of the picard problem we get

x	y	$y' = \frac{x^2}{y^2+1}$
$x_0 = 0$	$y_0 = 1$	$y_0' = \frac{0^2}{1^2+1} = 0$
$x_1 = 0.25$	$y_1 = 1.0026$	$y_1' = \frac{(0.25)^2}{(1.0026)^2+1} = 0.0312$
$x_2 = 0.5$	$y_2 = 1.0206$	$y_2' = \frac{(0.5)^2}{(1.0206)^2+1} = 0.1225$
$x_3 = 0.75$	$y_3 = 1.0679$	$y_3' = \frac{(0.75)^2}{(1.0679)^2+1} = 0.2628$
$x_4 = 1$	$y_4 = ?$	

$$y_4^{(P)} = y_3 + \frac{h}{24}(55y_3' - 59y_2' + 37y_1' - 9y_0')$$

$$= 1.0679 + \frac{0.25}{24}\left[55(0.2628) - 59(0.1225) + 37(0.0312) - 9(0)\right]$$

$$\boxed{y_4^{(P)} = 1.1552}$$

$x_4 = x_3 + h$

$h = x_4 - x_3 = 1 - 0.75$

$\boxed{h = 0.25}$

Now $y_4' = \frac{x_4^2}{y_4^2+1} = \frac{1^2}{(1.1552)^2+1} = 0.4284$

$$\boxed{y_4' = 0.4284}$$

Next $y_4^{(c)} = y_3 + \frac{h}{24}\left(9y_4' + 19y_3' - 5y_2' + y_1'\right)$ —— (1)

$= 1.0679 + \frac{0.25}{24}\left[9(0.4284) + 19(0.2628) - 5(0.1224) + 0.0312\right]$

$$\boxed{y_4^{(c)} = 1.154}$$

Now $y_4' = \frac{x_4^2}{y_4^2+1} = \frac{1^2}{(1.154)^2+1}$

$$\boxed{y_4' = 0.4289}$$

(1) $\Rightarrow$

$y_4^{(c)} = 1.1541$

Thus $\boxed{y(1) = 1.1541}$

7) Solve the di. eqn $y' + y + xy^2 = 0$ with the initial values of y : $y_0 = 1$, $y_1 = 0.9008$, $y_2 = 0.8066$, $y_3 = 0.722$ corresponding to the values of x: $x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$ by computing the value of y corresponding to $x = 0.4$ applying Adams - Bashforth p-c- formula (43)

we have the table,

x	y	$y' = -(y + xy^2)$
$x_0 = 0$	$y_0 = 1$	$y_0' = -1$
$x_1 = 0.1$	$y_1 = 0.9008$	$y_1' = +0.9819$
$x_2 = 0.2$	$y_2 = 0.8066$	$y_2' = -0.9367$
$x_3 = 0.3$	$y_3 = 0.722$	$y_3' = -0.8784$
$x_4 = 0.4$	$y_4 = ?$	

$x_4 = x_3 + h \Rightarrow h = x_4 - x_3 = 0.4 - 0.3$

$$\boxed{h = 0.1}$$

$$y_4^{(P)} = y_3 + \frac{h}{24}(55y_3' - 59y_2' + 37y_1' - 9y_0')$$

$$= 0.722 + \frac{0.1}{24}[55(-0.8784) - 59(+0.9367) + 37(-0.9819) - 9(-1)]$$

$$\boxed{y_4^{(P)} = 0.6371}$$

$$y_4' = -[y_4 + x_4 y_4^2]$$

$$\boxed{y_4' = -0.7995}$$

Next $y_4^{(c)} = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1')$ — (1)

$$= 0.722 + \frac{0.1}{24}\left(9(-0.7995) + 19(-0.8784) - 5(-0.9367) + (-0.9819)\right]$$

$$\boxed{y_4^{(c)} = 0.6379}$$

Now $y_4' = f(x_4, y_4)$

$$y_4' = -(y_4 + x_4 y_4^2)$$

$$= -(0.6379 + (0.4)(0.6379)^2)$$

$$\boxed{y_4' = -0.8007}$$

(1) $\Rightarrow$ $\boxed{y_4^{(c)} = 0.6379}$

Thus

$$\boxed{y(0.4) = 0.6379}$$