CBCS SCHEME

USN | | | | | | | | | | | **17MAT41**

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics - IV

Time: 3 hrs.   Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. From Taylor's series method, find y(0.1), considering upto fourth degree term if y(x) satisfying the equation $\frac{dy}{dx} = x - y^2$, y(0) = 1. **(06 Marks)**

b. Using Runge-Kutta method of fourth order $\frac{dy}{dx} + y = 2x$ at x = 1.1 given that y = 3 at x = 1 initially. **(07 Marks)**

c. If $\frac{dy}{dx} = 2e^x - y$, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040 and y(0.3) = 2.090, find y(0.4) correct upto four decimal places by using Milne's predictor-corrector formula. **(07 Marks)**

OR

2 a. Using modified Euler's method find y at x = 0.2 given $\frac{dy}{dx} = 3x + \frac{1}{2}y$ with y(0) = 1 taking h = 0.1. **(06 Marks)**

b. Given $\frac{dy}{dx} + y + zy^2 = 0$ and y(0) = 1, y(0.1) = 0.9008, y(0.2) = 0.8066, y(0.3) = 0.722. Evaluate y(0.4) by Adams-Bashforth method. **(07 Marks)**

c. Using Runge-Kutta method of fourth order, find y(0.2) for the equation $\frac{dy}{dx} = \frac{y - x}{y + x}$, y(0) = 1 taking h = 0.2. **(07 Marks)**

Module-2

3 a. Apply Milne's method to compute y(0.8) given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and the following table of initial values.

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y′	0	0.1996	0.3937	0.5689

(06 Marks)

b. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. **(07 Marks)**

c. Obtain the series solution of Bessel's differential equation $x^2y'' + xy' + (x^2 + n^2)\, y = 0$ leading to $J_n(x)$. **(07 Marks)**

OR

4 a. Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$, compute $y(0.2)$ and $y'(0.2)$ using fourth order Runge-Kutta method. **(06 Marks)**

b. Prove $J_{-1/2}(k) = \sqrt{\frac{2}{\pi x}} \cos x$. **(07 Marks)**

c. Prove the Rodfigues formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n y}{dx^n}(x^2 - 1)^n$ **(07 Marks)**

Module-3

5 a. Derive Cauchy-Riemann equations in Cartesian form. **(06 Marks)**

b. Discuss the transformation $w = z^2$. **(07 Marks)**

c. By using Cauchy's residue theorem, evaluate $\int_C \frac{e^{2z}}{(z+1)(z+2)} dz$ if C is the circle $|z| = 3$. **(07 Marks)**

OR

6 a. Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ **(06 Marks)**

b. State and prove Cauchy's integral formula. **(07 Marks)**

c. Find the bilinear transformation which maps $z = \infty, i, 0$ into $w = -1, -i, 1$. **(07 Marks)**

Module-4

7 a. Find the mean and standard of Poisson distribution. **(06 Marks)**

b. In an examination 7% of students score less than 35 marks and 89% of the students score less than 60 marks. Find the mean and standard deviation if the marks are normally distributed given A(1.2263) = 0.39 and A(1.4757) = 0.43 **(07 Marks)**

c. The joint probability distribution table for two random variables X and Y is as follows:

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine:
i) Marginal distribution of X and Y
ii) Covariance of X and Y
iii) Correlation of X and Y **(07 Marks)**

OR

8 a. A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
P(x)	0	K	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Find K and evaluate $P(x \geq 6)$, $P(3 < x \leq 6)$. **(06 Marks)**

b. The probability that a pen manufactured by a factory be defective is 1/10. If 12 such pens are manufactured, what is the probability that
i) Exactly 2 are defective
ii) Atleast two are defective
iii) None of them are defective. **(07 Marks)**

c. The length of telephone conversation in a booth has been exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made
i) Ends in less than 5 minutes
ii) Between 5 and 10 minutes. **(07 Marks)**

Module-5

9 a. A die is thrown 9000 times and a throw of 3 or 4 was observed 3240 times. Show that the dia cannot be regarded as an unbiased die. **(06 Marks)**

b. A group of 10 boys fed on diet A and another group of 8 boys fed on a different disk B for a period of 6 months recorded the following increase in weight (lbs):

Diet A:	5	6	8	1	12	4	3	9	6	10
Diet B:	2	3	6	8	10	1	2	8		

Test whether diets A and B differ significantly t.05 = 2.12 at 16df. **(07 Marks)**

c. Find the unique fixed probability vector for the regular stochastic matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$ **(07 Marks)**

OR

10 a. Define the terms:
i) Null hypothesis
ii) Type-I and Type-II error
iii) Confidence limits **(06 Marks)**

b. The t.p.m. of a Markov chain is given by $P = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \end{bmatrix}$. Find the fined probabilities vector. **(07 Marks)**

c. Two boys B_1 and B_2 and two girls G_1 and G_2 are throwing ball from one to another. Each boy throws the ball to the other boy with probability 1/2 and to each girl with probability 1/4. On the other hand each girl throws the ball to each boy with probability 1/2 and never to the other girl. In the long run how often does each receive the ball? **(07 Marks)**

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