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10MAT41

Fourth Semester B.E. Degree Examination, Dec.2015/Jan.2016

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: 1. *Answer FIVE full questions, selecting at least TWO questions from each part.*
2. *Use of statistical tables is permitted.*

PART – A

1 a. Using Taylor series method, solve the problem $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.2$. Consider upto 4^{th} degree terms. **(06 Marks)**

b. Using R.K. method of order 4, solve $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$ at the points $x = 0.1$ and $x = 0.2$ by taking step length $h = 0.1$. **(07 Marks)**

c. Given that $\frac{dy}{dx} = x - y^2$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. Compute y at $x = 0.8$ by Adams-Bashforth predictor-corrector method. Use the corrector formula twice. **(07 Marks)**

2 a. Evaluate y and z at $x = 0.1$ from the Picards second approximation to the solution of the following system of equations given by $y = 1$ and $z = 0.5$ at $x = 0$ initially.

$$\frac{dy}{dx} = z, \quad \frac{dz}{dx} = x^3(y + z)$$ **(06 Marks)**

b. Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$. Compute $y(0.2)$ and $y'(0.2)$ by taking $h = 0.2$ and using fourth order Runge-Kutta method. **(07 Marks)**

c. Applying Milne's method compute $y(0.8)$. Given that y satisfies the equation $y'' = 2yy'$ and y and y' are governed by the following values. $y(0) = 0$, $y(0.2) = 0.2027$, $y(0.4) = 0.4228$, $y(0.6) = 0.6841$, $y'(0) = 1$, $y'(0.2) = 1.041$, $y'(0.4) = 1.179$, $y'(0.6) = 1.468$. (Apply corrector only once). **(07 Marks)**

3 a. Derive Cauchy Riemann equations in Cartesian form. **(06 Marks)**

b. Find an analytic function $f(z) = u + iv$. Given $u = x^2 - y^2 + \frac{x}{x^2 + y^2}$. **(07 Marks)**

c. If f(z) is a regular function of z, show that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] |f(z)|^2 = 4\,|f'(z)|^2$ **(07 Marks)**

4 a. Find the bilinear transformation that maps the points $z = -1, i, -1$ onto the points $w = 1, i, -1$ respectively. **(06 Marks)**

b. Find the region in the w-plane bounded by the lines $x = 1$, $y = 1$, $x + y = 1$ under the transformation $w = z^2$. Indicate the region with sketches. **(07 Marks)**

c. Evaluate $\int_C \frac{e^{2z}}{(z+1)(z-2)} dz$ where c is the circle $|z| = 3$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – B

5 a. Solve the Laplaces equation in cylindrical polar coordinate system leading to Bessel differential equation. **(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = 0$ if $\alpha \neq \beta$. **(07 Marks)**

c. Express the polynomial, $2x^3 - x^2 - 3x + 2$ interms of Legendre polynomials. **(07 Marks)**

6 a. State and prove addition theorem of probability. **(06 Marks)**

b. Three students A, B, C write an entrance examination. Their chances of passing are ½, ⅓, ¼ respectively. Find the probability that,
   i) Atleast one of them passes.
   ii) All of them passes.
   iii) Atleast two of them passes. **(07 Marks)**

c. Three machines A, B, C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentages of defective outputs of these three machines are respectively 2%, 3% and 4%. An item is selected at random and is found to be defective. Find the probability that the item was produced by machine C. **(07 Marks)**

7 a. The pdf of a random variable x is given by the following table:

x	-3	-2	-1	0	1	2	3
P(x)	k	2k	3k	4k	3k	2k	k

Find: i) The value of k   ii) $P(x > 1)$ iii) $P(-1 < x \leq 2)$
iv) Mean of x   v) Standard deviation of x. **(06 Marks)**

b. In a certain factory turning out razar blades there is a small probability of 1/500 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing, i) One defective, ii) Two defective, in a consignment of 10000 packets. **(07 Marks)**

c. In a normal distribution 31% of items are under 45 and 8% of items are over 64. Find the mean and standard deviation of the distribution. **(07 Marks)**

8 a. A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 39350 kilometers with a standard deviation of 3260. Can it be considered as a true random sample from a population with mean life of 40000 kilometers? (Use 0.05 level of significance) Establish 99% confidence limits within which the mean life of tyres expected to lie. (Given that $Z_{0.05} = 1.96$, $Z_{0.01} = 2.58$) **(06 Marks)**

b. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. (Given that $t_{0.05} = 2.262$ for 9 d.f.) **(07 Marks)**

c. Fit a Poisson distribution to the following data and test the goodness of fit at 5% level of significance. Given that $\psi^2_{0.05} = 7.815$ for 4 degrees of freedom.

x	0	1	2	3	4
Frequency	122	60	15	2	1

(07 Marks)

* * * * *

Engineering Mathematics - IV

Dec 2015/Jan 2016

1.

a. $y' = x^2y - 1$, $y(0) = 1$, $y'(0) = -1$

$y'' = x^2 \cdot y' + y \cdot 2x$, $y''(0) = 0$

$y''' = x^2y'' + y' \cdot 2x + 2x \cdot y' + 2y$, $y_0'''(0) = 2$

$= x^2y'' + 4xy' + 2y$ $\boxed{2m}$

$y^{IV} = 2y' + 4y' + 4xy'' + 2xy'' + x^2y'''$

$= 6y' + 6xy'' + x^2y'''$ $\boxed{2m}$ $y^{IV}(0) = -6$

The Taylor's series expansion is given by

$$y(x) = y_0 + (x - x_0)y_0' + \frac{(x-x_0)^2}{2!} \cdot y_0'' + \frac{(x-x_0)^3}{3!} y_0''' + \cdots$$

$$= 1 + x \cdot (-1) + \frac{x^2}{2} \cdot (0) + \frac{x^3}{6} \times 2 + \frac{x^4}{4!}(-6)$$ $\boxed{1m}$

$\therefore y(0.2) = 0.8023$ $\boxed{1m}$

b. $y' = 3x + \frac{y}{2}$, $y(0) = 1$. $h = 0.1$

$k_1 = hf(x_0, y_0) = 0.1\, f(0, 1) = 0.05$

$k_2 = hf(x_0 + h/2, y_0 + \frac{k_1}{2}) = 0.1\, f[0.05, 1.025]$

$= 0.1[3(0.05) + \frac{1.025}{2}]$

$= 0.06625$ $\boxed{2m}$

$k_3 = hf[x_0 + h/2, y_0 + \frac{k_2}{2}] = 0.1\, f[0.05, 1.0331]$

$= 0.1[3(0.05) + 1.0331/2]$

$k_3 = 0.06667$, $k_4 = h f(x_0+h, y_0+k_3)$

$= 0.1 f(0.1, 1.0666) = 0.0833$

$y(0.1) = y_0 + 1/6 [k_1 + 2k_2 + 2k_3 + k_4] = 1.0665$ $\boxed{2m}$

2nd stage $\quad x_0 = 0.1$, $y_0 = 1.0665$

$k_1 = 0.0833$, $k_2 = .10041$, $k_3 = 0.10084$

$k_4 = 0.1183$, $y(0.2) = y_0 + 1/6 [k_1 + 2k_2 + 2k_3 + k_4]$

$= 1.16473$ $\boxed{3m}$

c. $y' = x - y^2$,

x	x_0 0	x_1 0.2	x_2 0.4	x_3 0.6	x_4 0.8
y	y_0 0	y_1 0.02	y_2 0.0795	y_3 0.1762	?
y'	0	0.1996	0.3937	0.5689	?

$\boxed{3m}$

$y_4^{(P)} = y_3 + \frac{h}{24}[55y_3' - 59y_2' + 37y_1' - 9y_0']$

$= 0.1762 + \frac{0.2}{24}[55(0.5689) - 59(0.3937) + 37(0.1996) - 9(0)]$

$= 0.3049$

$y_4' = x_4 - [y_4^{(P)}]^2 = 0.7071$ $\boxed{2m}$

$y_4^{(C)} = y_3 + \frac{h}{24}[9y_4' + 19y_3' - 5y_2' + y_1']$

$= 0.1763 + \frac{0.2}{24}[9(0.7071) + 19(0.5689) - 5(0.3937)$

$= 0.3049$

$y_4' = x_4 - [y_4^{(c)}]^2 = 0.7072$

$\therefore y_4^{(c)} = 0.3046$ (again applying corrector formula)

$\therefore y(0.8) = 0.3046$. $\boxed{2m}$

2. a. $y' = z, \quad z' = x^3(y+z), \quad y_0 = 1, z_0 = 0.5$
$x = 0$ $\boxed{1m}$

$y = 1 + \int_0^x z\,dx \quad -①, \quad z = \frac{1}{2} + \int_0^x x^3(y+z)\,dx \quad -②$ $\boxed{1m}$

Put $z = 1/2$ in ① & $y=1, z=1/2$ in ②

1st approx:

$y_1 = 1 + \int_0^x (1/2)\,dx, \quad z_1 = 1/2 + \int_0^x \frac{3}{2} x^3\,dx.$

$y_1 = 1 + x/2 \qquad z_1 = 1/2 + \frac{3x^4}{8}$ $\boxed{1m}$

2nd approx:

$y_2 = 1 + \int_0^x z_1\,dx \qquad z_2 = \frac{1}{2} + \int_0^x x^3(y_1+z_1)\,dx$

$y_2 = 1 + \frac{x}{2} + \frac{3x^5}{40} \qquad z_2 = 1/2 + \frac{3x^4}{8} + \frac{x^5}{10} + \frac{3x^8}{64}$ $\boxed{2m}$

$y(0.1) = 1.05, \quad z(0.1) = 0.5$ $\boxed{1m}$

b. $y' = z, \quad y'' - xy' - y = 0, \quad y_0 = 1, z_0 = 0$
$x_0 = 0.$

$y' = z, \quad z' = xz + y$ $\boxed{1m}$

$k_1 = h f(x_0, y_0, z_0) = 0.2\, f(0,1,0) = 0.$ $\boxed{1m}$
$l_1 = h g(x_0, y_0, z_0) = 0.2$

$$k_2 = hf[x_0 + h/2, y_0 + \tfrac{k_1}{2}, z_0 + l_1/2] = 0.02$$

$$l_2 = hg[x_0 + h/2, y_0 + \tfrac{k_1}{2}, z_0 + l_1/2] = 0.202$$

$\boxed{1m}$

$$k_3 = hf[x_0 + h/2, y_0 + \tfrac{k_2}{2}, z_0 + \tfrac{l_2}{2}] = 0.2 f(0.1, 1.01, 0.101)$$

$$= 0.0202 \quad \boxed{1m}$$

$$k_4 = hf[x_0 + h, y_0 + k_3, z_0 + l_3] = 0.2(0.204) = 0.0408$$

$$l_4 = hg[x_0 + h, y_0 + k_3, z_0 + l_3] = 0.2122 \quad \boxed{2m}$$

$$y(0.2) = 1.0202, \quad y'(0.2) = 0.204 \quad \boxed{1m}$$

c. $y' = z, \quad y'' = 2yy'$; $x_0 = 0, \ y_0 = 0, \ z_0 = 1$

x	0	0.2	0.4	0.6
y	0	0.2027	0.4228	0.6841
$y' = z$	1	1.041	1.179	1.468
$z' = 2yz$	0	0.422	0.997	2.009

$\boxed{3m}$

$$y_4^{(P)} = y_0 + \frac{4h}{3}[2z_1 - z_2 + 2z_3] \quad \boxed{1m}$$

$$= 0 + \frac{4(0.2)}{3}[2(1.041) - 1.179 + 2(1.468)]$$

$$= 1.0237$$

$$z_4^{(P)} = z_0 + \frac{4h}{3}[2z_1' - z_2' + 2z_3'] \quad \boxed{1m}$$

$$= 0 + \frac{4(0.2)}{3}[2(0.422) - 0.997 + 2(2.009)]$$

$$= 2.0307$$

$$z_4' = 2y_4^{(P)} z_4^{(P)} = 4.1571$$

$$y_4^{(c)} = y_2 + \frac{h}{3}\left[z_2 + 4z_3 + z_4 \right] = 1.0282$$

$$z_4^{(c)} = z_2 + h/3 \left[z_2' + 4z_3' + z_4' \right] = 2.0554$$

$$z_4' = 4.1577 \quad y(0.8) = 1.0301 \quad \boxed{2m}$$

3. a. Cauchy - Riemann Equations in Cartesian form.

Statement : The necessary conditions that the function $w = f(z) = u(x,y) + iv(x,y)$ may be analytic at any pt $z = x+iy$ is that, ∃ four continuous first order partial derivatives u_x, u_y, v_x, v_y & satisfy the equations $u_x = v_y$ & $v_x = -u_y$. These are known as C-R eqs. $\boxed{1m}$

Proof : Let $f(z)$ be analytic at a pt $z = x+iy$ & hence by the def

$$f'(z) = \lim_{\delta z \to 0} \frac{f(z+\delta z) - f(z)}{\delta z}$$ exists & is unique. $\boxed{1m}$

In the Cartesian form

$f(z) = u(x,y) + iv(x,y)$ & let δz be the increment in z corresponding to the increments $\delta x, \delta y$, in x, y.

$$f'(z) = \lt_{\delta z \to 0} \frac{[u(x+\delta x, y+\delta y) + iv(x+\delta x, y+\delta y) - u(x,y) + iv(x,y)]}{\delta z}$$

$$f'(z) = \lt_{\delta z \to 0} \frac{u(x+\delta x, y+\delta y) - u(x,y)}{\delta z} + i \lt_{\delta z \to 0} \frac{v(x+\delta x, y+\delta y) - v(x,y)}{\delta z} \quad -(1)$$

$\boxed{1m}$

$\delta z = \delta x + i\delta y$

$\because \delta z \to 0$, we have 2 possibilities

Case (1): Let $\delta y = 0 \Rightarrow \delta z = \delta x$

as $\delta z \to 0 \Rightarrow \delta x \to 0$.

(1) becomes

$$f'(z) = \lt_{\delta x \to 0} \frac{u(x+\delta x, y) - u(x,y)}{\delta x} + i \lt_{\delta x \to 0} \frac{v(x+\delta x, y) - v(x,y)}{\delta x}.$$

$f'(z) = u_x + iv_x \quad -(2)$ $\boxed{1m}$ (By basic def of Partial derivative)

Case (2): Let $\delta x = 0 \Rightarrow \delta z = i\delta y$

as $\delta z \to 0 \Rightarrow i\delta y \to 0$ ie $\delta y \to 0$

Now (1) becomes

$$f'(z) = \lt_{\delta y \to 0} \frac{u(x, y+\delta y) - u(x,y)}{i\delta y} +$$

$$i \lim_{\delta y \to 0} \frac{v(x, y+\delta y) - v(x,y)}{i\delta y}$$

$f'(z) = -iu_y + v_y$ — ③ $\boxed{1m}$ (By basic def of partial derivatives)

Comparing eqs ② & ③ we get

$u_x = v_y$ & $v_x = -u_y$. These are C-R eqs $\boxed{1m}$

6. $u = x^2 - y^2 + \dfrac{x}{x^2+y^2}$

$u_x = 2x + \dfrac{y^2 - x^2}{(x^2+y^2)^2}$ $\boxed{1m}$ $u_y = -2y - \dfrac{2xy}{(x^2+y^2)^2}$ $\boxed{1m}$

$f'(z) = u_x + iv_x$ $\boxed{1m}$

$= u_x - iu_y$ (By C-R eqs)

$= 2x + \dfrac{y^2 - x^2}{(x^2+y^2)^2} - i\left(-2y - \dfrac{2xy}{(x^2+y^2)^2}\right)$ $\boxed{1m}$

Put $x = z$, $y = 0$

$= 2z + \dfrac{-z^2}{(z^2)^2} - i\left(-\dfrac{(0)}{(z^2)^2}\right)$

$= 2z - \dfrac{1}{z^2}$ $\boxed{2m}$

on int.

$f(z) = z^2 + 1/z + c.$ $\boxed{1m}$

c. Let $f(z) = u + iv$ be analytic

$\therefore |f(z)| = \sqrt{u^2+v^2}$ or $|f(z)|^2 = u^2+v^2 = \phi$

To prove $\phi_{xx} + \phi_{yy} = 4|f'(z)|^2$

$\phi = u^2+v^2$, $\boxed{1m}$ $\qquad \phi_x = 2uu_x + 2vv_x$

$\qquad \phi_y = 2uu_y + 2vv_y$

$\phi_{xx} = 2[u u_{xx} + u_x^2 + v v_{xx} + v_x^2]$ — (1)

$|||^{ly}$ $\phi_{yy} = 2[u u_{yy} + u_y^2 + v v_{yy} + v_y^2]$ — (2)

adding (1) & (2) we get $\boxed{2m}$

$$\phi_{xx} + \phi_{yy} = 2[u(u_{xx}+u_{yy}) + v(v_{xx}+v_{yy}) + (u_x^2+u_y^2) + (v_x^2+v_y^2)]$$

as u & v are harmonic $u_{xx}+u_{yy} = v_{xx} + v_{yy} = 0$. By CR eqs $u_x = v_y$ & $v_x = -u_y$ $\boxed{1m}$

$$\therefore \phi_{xx}+\phi_{yy} = 2[(u_x^2+v_x^2) + (v_x^2+u_x^2)]$$

$$= 2[2(u_x^2+v_x^2)]$$

$$= 4[u_x^2+v_x^2] \quad \boxed{2m}$$

$|f'(z)| = \sqrt{u_x^2+v_x^2}$, $|f'(z)|^2 = u_x^2+v_x^2$

$\therefore \phi_{xx} + \phi_{yy} = 4|f'(z)|^2$ $\boxed{1m}$

4.

a. $z = 1, i, -1$, $w = 1, i, -1$

$w = \dfrac{az+b}{cz+d}$ $\quad ad-bc \neq 0$ $\boxed{1m}$

$$\frac{(w_1-w_1)(w_2-w_3)}{(w_2-w_3)(w_2-w_1)} = \frac{(z_1-z_1)(z_2-z_3)}{(z_2-z_3)(z_2-z_1)}$$ 2m

$$\frac{(w-1)(i+1)}{(w+1)(i-1)} = \frac{(z+1)(i+1)}{(z+1)(i+1)}$$

$$\Rightarrow \frac{w-1}{w+1} = \frac{i-1}{i+1} \Rightarrow \frac{w-1}{w+1} = \frac{i-1}{i+1} \times \frac{i-1}{i-1} = i$$ 2m

$$\Rightarrow w-1 = i(w+1) \Rightarrow w(1-i) = (i+1)$$

$$\Rightarrow w = \frac{i+1}{i-1}$$ 1m

b. $w = z^2$

$u + iv = (x+iy)^2 = (x^2-y^2) + 2ixy$

$u = x^2 - y^2$, $v = 2xy$ — ① 1m

consider $x=1$, ① becomes $u = 1-y^2$, $v = 2y$

substitute $v/2 = y$ in u we have $u = 1 - v^2/4$

ie $v^2 = 4(1-u)$. This is parabola in w-plane with vertex $(1,0)$ & symmetrical about the u-axis. 1m

consider $y=1$. ① becomes $u = x^2-1$, $v = 2x$

sub $v/2 = x$ in u we have $u = v^2/4 - 1$

$v^2 = 4(1+u)$. This is also a parabola in the w-plane with vertex $(-1,0)$ & symmetrical about the u-axis. 2m

Consider $x+y=1$ or $y = 1-x$.

① ~~becomes~~ $u = x^2 - (1-x)^2$

$u = -1 + 2x, \quad v = 2x(1-x)$

Substituting $2x = 1+u, \quad x = \frac{1}{2}(1+u)$, v becomes

$v = (1+u)\left(1 - \frac{1+u}{2}\right) = (1+u)\left(\frac{1-u}{2}\right)$ ~~or~~

$v = \frac{1}{2}(1-u^2)$ $\boxed{1m}$

ie $1 - u^2 = 2v, \quad u^2 = -2\left[v - \frac{1}{2}\right]$. This is also a parabola in the w-plane with vertex $(0, \frac{1}{2})$ symmetrical about the v-axis.

z-plane

$\boxed{2m}$

C. $I = \int_C \frac{e^{2z}}{(z+1)(z-2)} dz \qquad |z| = 3$

Sol: The points $z = a = -1, \; z = a = 2$ being $(-1,0), (2,0)$ lies inside $|z| = 3$

$$\frac{1}{(z+1)(z-2)} = \frac{A}{z+1} + \frac{B}{z-2}$$ $\boxed{1m}$

Resolving into partial fractions

$A = -\frac{1}{3}, \quad B = \frac{1}{3}$ $\boxed{2m}$

$$\frac{e^{2z}}{(z+1)(z-2)} = -\frac{1}{3}\frac{1}{z+1} + \frac{1}{3}\frac{1}{z-2}$$

$$\int_C \frac{e^{2z}}{(z+1)(z-2)} = \frac{1}{3}\left[\frac{e^{2z}}{z-2} - \frac{e^{2z}}{z+1}\right]$$

$$\int_C \frac{e^{2z}}{(z+1)(z-2)}\,dz = \frac{1}{3}\left[\int_C \frac{e^{2z}}{z-2}\,dz - \int_C \frac{e^{2z}}{z+1}\,dz\right] \quad \boxed{1m}$$

$$\int_C \frac{f(z)\,dz}{z-a} = 2\pi i\, f(a) \quad \boxed{1m}$$ (By Cauchy's integral formula)

$$\int_C \frac{e^{2z}\,dz}{(z+1)(z-2)} = \frac{1}{3}\left[2\pi i e^4 - \frac{2\pi i}{e^2}\right]$$

$$\int_C \frac{e^{2z}}{(z+1)(z-2)}\,dz = \frac{2\pi i}{3}\left[e^4 - \frac{1}{e^2}\right] \quad \boxed{2m}$$

5.

a. The coordinates (ρ, ϕ, z) are called cylindrical coordinates by the relationship with the Cartesian coordinates (x, y, z) is given by $x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$.

The Laplace eq $\nabla^2 f = 0$ in the cylindrical system is given by

$$\frac{\partial^2 f}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial f}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} = 0 \quad \boxed{1m} \quad -(1)$$

We shall solve this by the method of separation of variables.

Let $f = f_1 f_2 f_3$ be the sol of (1) where

$f_1 = f_1(\rho)$, $f_2 = f_2(\phi)$, $f_3 = f_3(z)$ $\boxed{1m}$

$\therefore$ (1) becomes

$$\frac{\partial^2 f_1 f_2 f_3}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial (f_1 f_2 f_3)}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 f_1 f_2 f_3}{\partial \phi^2} + \frac{\partial^2 f_1 f_2 f_3}{\partial z^2} = 0$$

÷ by $f_1 f_2 f_3$

$$\frac{1}{f_1}\frac{d^2 f_1}{d\rho^2} + \frac{1}{\rho f_1}\frac{d f_1}{d\rho} + \frac{1}{\rho^2 f_2}\frac{d^2 f_2}{d\phi^2} + \frac{1}{f_3}\frac{d^2 f_3}{dz^2} = 0$$

$$\frac{1}{f_1}\frac{d^2 f_1}{d\rho^2} + \frac{1}{\rho f_1}\frac{d f_1}{d\rho} + \frac{1}{\rho^2 f_2}\frac{d^2 f_2}{d\phi^2} = -\frac{1}{f_3}\frac{d^2 f_3}{dz^2} \quad -(2)$$

Let us set $\frac{1}{f_3}\frac{d^2 f_3}{dz^2} = 1$

$\boxed{1m}$

∴ (2) becomes

$$\frac{1}{f_1}\frac{d^2 f_1}{d\rho^2} + \frac{1}{\rho f_1}\frac{d f_1}{d\rho} + \frac{1}{\rho^2 f_2}\frac{d^2 f_2}{d\phi^2} = -1$$

Now × by ρ^2 we get.

$$\frac{\rho^2}{f_1}\frac{d^2 f_1}{d\rho^2} + \frac{\rho}{f_1}\frac{d f_1}{d\rho} + \frac{1}{f_2}\frac{d^2 f_2}{d\phi^2} = -\rho^2$$

$$\frac{\rho^2}{f_1}\frac{d^2 f_1}{d\rho^2} + \frac{\rho}{f_1}\frac{d f_1}{d\rho} + \rho^2 = -\frac{1}{f_2}\frac{d^2 f_2}{d\phi^2}$$

$\boxed{1m}$

Again L.H.S is a func of ρ & R.H.S is a func of ϕ. ∴ $-\frac{1}{f_2}\frac{d^2 f_2}{d\phi^2} = n^2$

$$\therefore \frac{\rho^2}{f_1}\frac{d^2 f_1}{d\rho^2} + \frac{\rho}{f_1}\frac{d f_1}{d\rho} + \rho^2 = n^2$$

$$\frac{\rho^2}{f_1}\frac{d^2 f_1}{d\rho^2} + \frac{\rho}{f_1}\frac{df_1}{d\rho} + (\rho^2 - n^2) = 0$$

$$\rho^2 \frac{d^2 f_1}{d\rho^2} + \rho \frac{df_1}{d\rho} + (\rho^2 - n^2) f_1 = 0 \quad \boxed{1m}$$

This eq can be written in the form

$$x^2 y'' + xy' + (x^2 - n^2) y = 0 \quad \boxed{1m}$$

This is the Bessel's differential eq of order n.

b. If α & β are 2 distinct roots of $J_n(x) = 0$ then $\int_0^1 x J_n(\alpha x) J_n(\beta x)\, dx = 0$. if $\alpha \neq \beta$ $\boxed{1m}$

Proof: W.K.T $J_n(\lambda x)$ is a solution of the eq

$$x^2 y'' + xy' + (\lambda^2 x^2 - n^2) y = 0$$

If $u = J_n(\alpha x)$ & $v = J_n(\beta x)$ the associated differential eqs are

$$x^2 u'' + xu' + (\alpha^2 x^2 - n^2) u = 0 \quad -(1)$$

$$x^2 v'' + xv' + (\beta^2 x^2 - n^2) v = 0 \quad -(2)$$

x'ly (1) by $\frac{v}{x}$ & (2) by $\frac{u}{x}$

$$xvu'' + vu' + \alpha^2 uvx - \frac{n^2 uv}{x} = 0$$

$$xuv'' + uv' + \beta^2 uvx - \frac{n^2 uv}{x} = 0 \quad \boxed{3m}$$

on subtracting we obtain

$$x(vu''-uv'')+(vu'-uv')+(\alpha^2-\beta^2)uvx=0$$

$$\frac{d}{dx}\{x(vu'-uv')\}_{x=0}^{1}=(\beta^2-\alpha^2)\int_0^1 xuv\,dx$$

$$(vu'-uv')_{x=1}=0=(\beta^2-\alpha^2)\int_0^1 xuv\,dx \quad -③$$

$\because u=J_n(\alpha x)$ $v=J_n(\beta x)$ we have $u'=\alpha J_n'(\alpha x)$

$v'=\beta J_n'(\beta x)$ & as a consequence of ③ becomes

$$\left[J_n(\beta x)\,\alpha J_n'(\alpha x)-J_n(\alpha x)\,\beta J_n'(\beta x)\right]_{x=1}$$

$$=(\beta^2-\alpha^2)\int_0^1 x\,J_n(\alpha x)J_n(\beta x)\,dx$$

Hence $\int_0^1 x\,J_n(\alpha x)J_n(\beta x)\,dx=\frac{1}{\beta^2-\alpha^2}\left(\alpha J_n(\beta)J_n'(\alpha)-\beta J_n(\alpha)J_n'(\beta)\right)$ $-④$

$\because \alpha$ & β are distinct roots $\boxed{2m}$ of $J_n(x)=0$

we have $J_n(\alpha)=0$ & $J_n(\beta)=0$ with the result the R.H.S of ④ becomes zero

provided $\beta^2-\alpha^2\neq 0$ or $\beta\neq\alpha$.

Thus we have proved that if $\alpha\neq\beta$.

$$\int_0^1 x\,J_n(\alpha x)\,J_n(\beta x)\,dx=0. \quad \boxed{1m}$$

C. $2x^3-x^2-3x+2$.

$$x^3 = \frac{2}{5}P_3(x) + \frac{3}{5}P_1(x), \quad x^2 = \frac{1}{3}P_0(x) + \frac{2}{3}P_2(x)$$

$$x = P_1(x) \ldots, \quad 1 = P_0(x)$$

$\boxed{3m}$

$$2x^3 - x^2 - 3x + 2 = 2\left[\frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)\right] - \left[\frac{1}{3}P_0(x) + \frac{2}{3}P_2(x)\right] - 3P_1(x) + 2P_0(x)$$

$$= \frac{4}{5}P_3(x) + \frac{6}{5}P_1(x) - \frac{1}{3}P_0(x) - \frac{2}{3}P_2(x) - 3P_1(x) + 2P_0(x)$$

$\boxed{2m}$

$$= \frac{4}{5}P_3(x) - \frac{2}{3}P_2(x) + P_1(x)\left(-\frac{9}{5}\right) + \frac{5}{3}P_0(x)$$

$\boxed{2m}$

6.

a. Addition theorem of Probability

The probability of the happening of one or the other mutually exclusive events is equal to the sum of the probabilities of the two events. i.e If A, B are 2 mutually exclusive events then

$$P(A \text{ or } B) = P(A) + P(B)$$

$\boxed{2m}$

Proof: Let the total number of exhaustive, mutually exclusive & equally possible cases in the trials be n. Out of these

m_1 cases be favourable to the event A

& m_2 cases be favourable to B.

Hence the no of cases favourable to either A or B is $m_1 + m_2$

$\therefore P(A \text{ or } B) = \frac{m_1 + m_2}{n} = \frac{m_1}{n} + \frac{m_2}{n}$ — ① $\boxed{2m}$

$\because$ m_1 cases are favourable to A, $P(A) = \frac{m_1}{n}$

$\because$ m_2 cases are favourable to B, $P(B) = \frac{m_2}{n}$.

Substituting in R.H.S of ①

$P(A \text{ or } B) = P(A) + P(B)$. $\boxed{2m}$

b. $P(A) = 1/2$, $P(B) = 1/3$, $P(C) = 1/4$

$P(\bar{A}) = 1/2$, $P(\bar{B}) = 2/3$, $P(\bar{C}) = 3/4$ $\boxed{2m}$

i) atleast one of them passes

$P(A \cup B \cup C) = 1 - [P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})]$

$= 1 - (1/2 \cdot 2/3 \cdot 3/4) = 3/4$ $\boxed{1m}$

ii) All of them passes

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$

$= 1/24$. $\boxed{1m}$

iii) Atleast two of them passess

$\therefore P(A) \cdot P(B) \cdot P(\bar{C}) + P(A) P(\bar{B}) \cdot P(C) + P(\bar{A}) \cdot P(B) \cdot P(C)$

$+ P(A) \cdot P(B) \cdot P(C)$

$= \frac{7}{24}$. $\boxed{3m}$

6c. $P(A) = 0.6$, $P(B) = 0.3$, $P(C) = 0.1$ $\boxed{1m}$

Let E be the event of defective output from 3 machines.

$P(E/A) = 0.02$, $P(E/B) = 0.03$ $\boxed{2m}$

$P(E/C) = 0.04$

To find $P(C/E)$

By Baye's theorem for conditional Prob we have. $\boxed{1m}$

$$P(C/E) = \frac{P(C) \cdot P(E/C)}{P(A) \cdot P(E/A) + P(B) \cdot P(E/B) + P(C) \cdot P(E/C)}$$

$= 0.16$. $\boxed{3m}$

7.

a. i) The value of k. $\Sigma P(x) = 1$

$k + 2k + 3k + 4k + 3k + 2k + k = 1$

$\Rightarrow k = 1/16$ $\boxed{1m}$

b. $P(x>1) = P(2) + P(3)$

$= \frac{2}{16} + \frac{1}{16} = \frac{3}{16}$ $\boxed{1m}$

c. $P(-1 < x \le 2)$

$P(0) + P(1) + P(2)$

$\frac{4}{16} + \frac{3}{16} + \frac{2}{16}$

$= \frac{9}{16}$ $\boxed{1m}$

d. Mean of x.

$\Sigma x\, p(x) = 0$ $\boxed{1m}$

e. variance

$V = \Sigma (x_i - \mu)^2 p(x_i)$

$= \frac{5}{2}$.

$S.D = \sqrt{V} = \sqrt{5/2} = 1.5811$ $\boxed{2m}$

b.

$P = 1/500 = 0.002$ Mean $\lambda = np = 0.02$ $\boxed{2m}$

$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ $\boxed{1m}$

i) One defective : $10{,}000 \times P(x) = 10000 \times \frac{\lambda^1 e^{-\lambda}}{1!}$

$= 196$ $\boxed{2m}$

ii) Two defective . $10{,}000 \times P(x) = 10000 \times \frac{\lambda^2 e^{-\lambda}}{2!}$

$= 2$. $\boxed{2m}$

c. Let μ & σ be the mean & S.D of the normal distribution

$P(x<35) = 0.07$, $P(x < 60) = 0.89$ $\boxed{1m}$

S.n.v =) $Z = \frac{x - \mu}{\sigma}$

when $x = 35$, $Z = \frac{35 - \mu}{\sigma} = Z_1$.

when $x = 60$ $\quad Z = \frac{60 - \mu}{\sigma} = Z_2$. $\boxed{2m}$

$P(Z < Z_1) = 0.07$, $P(Z < Z_2) = 0.89$

$0.5 + \phi(z_1) = 0.07$, $\quad 0.5 + \phi(Z_2) = 0.89$

$\phi(z_1) = -0.43$, $\quad \phi(Z_2) = 0.39$

$\phi(z_1) = -\phi(1.4757)$, $\phi(Z_2) = \phi(1.2263)$

$Z_1 = -1.4757$, $Z_2 = 1.2263$

$\frac{35 - \mu}{\sigma} = -1.4757$, $\frac{60 - \mu}{\sigma} = 1.2263$ $\boxed{3m}$

$\mu - 1.4757\sigma = 35$, $\mu + 1.2263\sigma = 60$

By solving $\mu = 48.65$, $\sigma = 9.25$. $\boxed{1m}$

8.

a. Assume ~~the~~ null hypothesis

$H_0 = \mu = 40,000$ & alternate hypothesis

$H: \mu \neq 40,000$ $\boxed{1m}$

$\bar{x} = 39,350$, $\mu = 40000$, $\sigma = 3260$ $\boxed{1m}$

$n = 100$.

$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = 1.9947 \quad Z_{0.05} = 1.96$ $\boxed{1m}$

$\therefore H_0$ is rejected i.e we cannot say that it is a true sample from a population with mean = 40,000 . Now 99% confidence limits within which the population mean is expected to lie is given as $\bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}} = (38509, 40{,}191)$ $\boxed{3m}$

b. $\bar{x} = \frac{\Sigma x}{n} = 67.8$ $\boxed{2m}$, $\mu = 66$ $\boxed{1m}$, $s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = 9.06795 \quad s = 3.011$ $\boxed{2m}$

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \quad 1.89 < t_{0.05} = 2.261$ $\boxed{1m}$

$\therefore$ The hypothesis is accepted at 5% level of significance. $\boxed{1m}$

c. $\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = 0.5$ $\boxed{2m}$ we take this as the mean of the poisson distribution

$\lambda = 0.5$

Hence expected frequency are given by

$E_i = \frac{N e^{-\lambda} \lambda^x}{x!}$ $\boxed{1m}$

$N = 200, \; x = 0, 1, 2, 3, 4$

O_i 122, 60, 15, 2, 1. $\boxed{2m}$

E_i 121, 61, 15, 3, 0

$\chi^2 = \frac{(O_i - E_i)^2}{E_i} = 0.025$ $\boxed{1m}$

This is less than $\chi^2_{0.05} = 7.815$

Hence fitness is considered good. $\boxed{1m}$