USN 1CR12IS125

Fourth Semester B.E. Degree Examination, June/July 2014

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain a solution upto the third approximation of y for x = 0.2 by Picard's method, given that $\frac{dy}{dx} + y = e^x$; $y(0) = 1$. **(06 Marks)**

b. Apply Runge-Kutta method of order 4, to find an approximate value of y for x = 0.2 in steps of 0.1, if $\frac{dy}{dx} = x + y^2$ given that y = 1 when x = 0. **(07 Marks)**

c. Using Adams-Bashforth formulae, determine y(0.4) given the differential equation $\frac{dy}{dx} = \frac{1}{2}xy$ and the data, y(0) = 1, y(0.1) = 1.0025, y(0.2) = 1.0101, y(0.3) = 1.0228. Apply the corrector formula twice. **(07 Marks)**

2 a. Apply Picard's method to find the second approximation to the values of 'y' and 'z' given that $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^3(y+z)$, given $y = 1, z = \frac{1}{2}$ when x = 0. **(06 Marks)**

b. Using Runge-Kutta method, solve $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0$ for x = 0.2 correct to four decimal places. Initial conditions are x = 0, y = 1, y′ = 0. **(07 Marks)**

c. Obtain the solution of the equation $\frac{2d^2y}{dx^2} = 4x + \frac{dy}{dx}$ at the point x = 1.4 by applying Milne's method given that y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649. y(1.3) = 2.7514, y′(1) = 2, y′(1.1) = 2.3178, y′(1.2) = 2.6725 and y′(1.3) = 3.0657. **(07 Marks)**

3 a. Define an analytic function in a region R and show that f(z) is constant, if f(z) is an analytic function with constant modulus. **(06 Marks)**

b. Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2 + y^2}$ are harmonic functions of (x, y) but are not harmonic conjugate. **(07 Marks)**

c. Determine the analytic function f(z) = u + iv, if $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ and $f(\pi/2) = 0$. **(07 Marks)**

4 a. Find the images of the circles |z| = 1 and |z| = 2 under the conformal transformation $w = z + \frac{1}{z}$ and sketch the region. **(06 Marks)**

b. Find the bilinear transformation that transforms the points 0, i, ∞ onto the points 1, –i, –1 respectively. **(07 Marks)**

c. State and prove Cauchy's integral formula and hence generalized Cauchy's integral formula. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – B

5 a. Obtain the solution of the equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-\frac{1}{4}\right)y=0$. **(06 Marks)**

b. Obtain the series solution of Legendre's differential equation,

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$ **(07 Marks)**

c. State Rodrigue's formula for Legendre polynomials and obtain the expression for $P_4(x)$ from it. Verify the property of Legendre polynomials in respect of $P_4(x)$ and also find $\int_{-1}^{1}x^3P_4(x)dx$. **(07 Marks)**

6 a. Two fair dice are rolled. If the sum of the numbers obtained is 4, find the probability that the numbers obtained on both the dice are even. **(06 Marks)**

b. Given that $P(\overline{A}\cap\overline{B})=\frac{7}{12}$, $P(A\cap\overline{B})=\frac{1}{6}=P(\overline{A}\cap B)$. Prove that A and B are neither independent nor mutually disjoint. Also compute $P(A/B)+P(B/A)$ and $P(\overline{A}/\overline{B})+P(\overline{B}/\overline{A})$. **(07 Marks)**

c. Three machines M_1, M_2 and M_3 produces identical items. Of their respective outputs 5%, 4% and 3% of items are faulty. On a certain day, M_1 has produced 25% of the total output, M_2 has produced 30% and M_3 the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with the highest output? **(07 Marks)**

7 a. In a quiz contest of answering 'Yes' or 'No', what is the probability of guessing atleast 6 answers correctly out of 10 questions asked? Also find the probability of the same if there are 4 options for a correct answer. **(07 Marks)**

b. Define exponential distribution and obtain the mean and standard deviation of the exponential distribution. **(07 Marks)**

c. If X is a normal variate with mean 30 and standard deviation 5, find the probabilities that (i) $26\le X\le 40$, (ii) $X\ge 45$, (iii) $|X-30|>5$. [Give that $\phi(0.8)=0.2881$, $\phi(2.0)=0.4772$, $\phi(3.0)=0.4987$, $\phi(1.0)=0.3413$] **(06 Marks)**

8 a. Certain tubes manufactured by a company have mean life time of 800 hrs and standard deviation of 60 hrs. Find the probability that a random sample of 16 tubes taken from the group will have a mean life time (i) between 790 hrs and 810 hrs, (ii) less than 785 hrs, (iii) more than 820 hrs. [$\phi(0.67)=0.2486$, $\phi(1)=0.3413$, $\phi(1.33)=0.4082$]. **(06 Marks)**

b. A set of five similar coins is tossed 320 times and the result is:

No. of heads:	0	1	2	3	4	5
Frequency:	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution. [Given that $\psi^2_{0.05}(5)=11.07$] **(07 Marks)**

c. It is required to test whether the proportion of smokers among students is less than that among the lectures. Among 60 randomly picked students, 2 were smokers. Among 17 randomly picked lecturers, 5 were smokers. What would be your conclusion? **(07 Marks)**

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Engineering Mathematics - IV

June/July 2014

PART - A

1. a) Obtain a Solution upto the third approximation of y for $x=0.2$ by Picard's method, given that $\frac{dy}{dx}+y=e^x$ and $y(0)=1$.

Sol:- given:- $\frac{dy}{dx}+y=e^x$

$$dy=(e^x-y)dx \quad ; \quad y=1,\ x=0$$

Integrating we get

$$\int_1^y dy=\int_0^x (e^x-y)dx \Rightarrow y-1=\int_0^x (e^x-y)dx$$

$$\therefore \quad y=1+\int_0^x (e^x-y)dx$$

now

$$y_1=1+\int_0^x (e^x-1)dx$$

$$=1+[e^x-x]_0^x=1+e^x-x-1$$

$$y_1=e^x-x$$

$$y_2=1+\int_0^x (e^x-y_1)dx$$

$$=1+\int_0^x [e^x-(e^x-x)]dx$$

$$=1+\left[\frac{x^2}{2}\right]_0^x=1+\frac{x^2}{2}$$

$$y_3=1+\int_0^x (e^x-y_2)dx$$

$$=1+\int_0^x \left[e^x-\left(1+\frac{x^2}{2}\right)\right]dx$$

$$= 1 + \left[e^x - x - \frac{x^3}{6} \right]_0^x$$

$$\therefore \; y_3 = e^x - x - \frac{x^3}{6}$$

$$\therefore \; y_3(0.2) = 1.0200694$$

b) Apply R-K method of order 4, to find an approx Value of y for $x=0.2$ in steps of 0.1, if $\frac{dy}{dx} = x+y^2$ given that $y=1$ when $x=0$

Sol:- Stage:- I:- $f(x,y) = x+y^2$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$$K_1 = hf(x_0, y_0) = 0.1 f(0,1) = 0.1$$

$$K_2 = hf\left(x_0+\frac{h}{2}, y_0+\frac{K_1}{2}\right) = 0.1 f(0.05, 1.05) = 0.11525$$

$$K_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}\right) = 0.1 f(0.05, 1.0576) = 0.11685$$

$$K_4 = hf(x_0+h, y_0+K_3) = 0.1 f(0.1, 1.11685) = 0.134765$$

$$y(x_0+h) = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$y(0.1) = 1 + \frac{1}{6}(0.1 + 0.2305 + 0.2337 + 0.1347)$$

$$y(0.1) = 1.1165$$

Stage II:- $f(x,y) = x+y^2$, $x_0 = 0.1$, $y_0 = 1.1165$, $h = 0.1$

$$K_1 = 0.1 f(0.1, 1.1165592) = 0.1346$$

$$K_2 = hf\left(x_0+\frac{h}{2}, y_0+\frac{K_1}{2}\right) = 0.1 f(0.15, 1.18383) = 0.1551$$

$$K_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}\right) = 0.1 f(0.15, 1.19417) = 0.15757$$

$$K_4 = hf(x_0+h, y_0+K_3) = 0.1 f(0.2, 1.27409) = 0.1823$$

$$y(0.2) = y_0 + \frac{1}{6}(0.1301 + 0.2994 + 0.30414 + 0.1573)$$

$$y(0.2) = 1.2736$$

c) Using Adams–Bashforth formulae, determine $y(0.4)$ given the differential eq^n $\frac{dy}{dx} = \frac{1}{2}xy$ and the data $y(0) = 1$, $y(0.1) = 1.0025$, $y(0.2) = 1.0101$, $y(0.3) = 1.0228$. Apply Corrector formula twice

Sol:-

x	y	$y' = \frac{1}{2}xy$
$x_0 = 0$	$y_0 = 1$	$y_0' = 0$
$x_1 = 0.1$	$y_1 = 1.0025$	$y_1' = 0.050125$
$x_2 = 0.2$	$y_2 = 1.0101$	$y_2' = 0.1010$
$x_3 = 0.3$	$y_3 = 1.0228$	$y_3' = 0.15342$
$x_4 = 0.4$	$y_4 = ?$	

now $y_4^{(P)} = y_3 + \frac{h}{24}[55y_3' - 59y_2' + 37y_1' - 9y_0']$

$= 1.0228 + \frac{(0.1)}{24}(55 \times 0.1534 - 59 \times 0.1010 + 37 \times 0.0501 - 0)$

$y_4 = 1.040812$

$\Rightarrow y_4' = \frac{1}{2}(0.4)(1.0408) = 0.20816$

now $y_4^{(C)} = y_3 + \frac{h}{24}[9y_4' + 19y_3' - 5y_2' + y_1']$

$= 1.0228 + \frac{0.1}{24}[9 \times .20818 + 19 \times .1534 - 5 \times 0.1010 + 0.0501]$

$y_4^{(C)} = 1.04086$

$\Rightarrow y_4' = \frac{1}{2}(0.4)(1.0408) = 0.20816$

Again Apply Corrector's formula we get

$y_4^{(C)} = y(0.4) = 1.04086$

$y_4' = ~~1.040~~\ 0.20816.$

2 a) Apply Picard's method to find the second approximation to the values of 'y' and 'z' given that $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^3(y+z)$, given $y=1$, $z=\frac{1}{2}$ when $x=0$.

Sol.

We have by data,

$$dy = z\,dx;\ y=1,\ x=0;\ dz = x^3(y+z)\,dx;\ z=\frac{1}{2},\ x=0$$

Now $\int_1^y dy = \int_0^x z\,dx$; $\int_{1/2}^z dz = \int_0^x x^3(y+z)\,dx$

Hence, $y = 1 + \int_0^x z\,dx$ --- (1)

$$z = \frac{1}{2} + \int_0^x x^3(y+z)\,dx \quad --- (2)$$

Putting $z=1/2$ in the RHS of (1) and $y=1, z=\frac{1}{2}$ in the RHS of (2) we have,

$$y_1 = 1 + \int_0^x \frac{1}{2}\,dx \quad ; \quad z_1 = \frac{1}{2} + \int_0^x \frac{3}{2}x^3\,dx$$

$$\therefore y_1 = 1 + \frac{x}{2} \quad ; \quad z_1 = \frac{1}{2} + \frac{3x^4}{8}$$

Now, $y_2 = 1 + \int_0^x z_1\,dx$; $z_2 = \frac{1}{2} + \int_0^x x^3(y_1+z_1)\,dx$

$$y_2 = 1 + \int_0^x \left(\frac{1}{2} + \frac{3x^4}{8}\right)dx \quad ; \quad z_2 = \frac{1}{2} + \int_0^x x^3\left(\frac{3}{2} + \frac{x}{2} + \frac{3x^4}{8}\right)dx$$

$$\therefore y_2 = 1 + \frac{x}{2} + \frac{3x^5}{40} \quad ; \quad z_2 = \frac{1}{2} + \frac{3x^4}{8} + \frac{x^5}{10} + \frac{3x^8}{64}$$

2) b) Using Runge-Kutta method, solve $\frac{d^2y}{dx} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0$ for $x=0.2$ correct to 4 decimal places. Initial conditions are $x=0$ $y=1$ $y'=0$.

<u>Sol.</u>

By data. $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$

putting $\frac{dy}{dx} = z$ and differentiating w.r.t x, we obtain $\frac{d^2y}{dx^2} = \frac{dz}{dx}$

The given equation becomes,

$\frac{dz}{dx} = xz^2 - y^2$ with $y=1, z=0$ when $x=0$

Hence, we have a system of equations $\frac{dy}{dx} = z$, $\frac{dz}{dx} = xz^2 - y^2$

Let $f(x,y,z) = z$, $g(x,y,z) = xz^2 - y^2$, $x_0 = 0$, $y_0 = 1$, $z_0 = 0$ & $h = 0.2$

We shall first compute the following.

$k_1 = hf(x_0, y_0, z_0) = (0.2) f(0,1,0) = (0.2)(0) = 0$

$l_1 = (0.2)\left[(0)(0)^2 - (1)^2\right] = -0.2$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$

$k_2 = (0.2) f(0.1, 1, -0.1) = (0.2)(-0.1) = -0.02$

$l_2 = (0.2)\left[(0.1)(0.1)^2 - (1)^2\right] = -0.1998$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$

$k_3 = (0.2) f(0.1, 0.99, -0.0999) = (0.2)(0.0999) = -0.01998$

$l_3 = (0.2)\left[(0.1)(-0.0999)^2 - (0.99)^2\right] = -0.1958$

$k_4 = hf(x_0 + h, y_0 + k_3, z_0 + l_3)$

$k_4 = (0.2) f(0.2, 0.98002, -0.1958) = (0.2)(-0.1958) = -0.03916$

$$k_4 = (0.2)\left[(0.2)(-0.1958)^2 - (0.98002)^2\right] = -0.19055$$

We have $y(x_0+h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$$y(0.2) = 1 + \frac{1}{6}\left[0 + 2(-0.02) + 2(-0.01998) - 0.03916\right]$$

Thus $y(0.2) = 0.9801$

c) Obtain the solution of the equation $\frac{2d^2y}{dx^2} = 4x + \frac{dy}{dx}$ at the point $x = 1.4$ by applying Milne's method given that $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$. $y(1.3) = 2.7514$, $y'(1) = 2$, $y'(1.1) = 2.3178$, $y'(1.2) = 2.6725$ and $y'(1.3) = 3.0657$.

Sol.

Dividing the given equation by 2 we have,

$$\frac{d^2y}{dx^2} = 2x + \frac{1}{2}\frac{dy}{dx} \quad \text{or} \quad y'' = 2x + \frac{y'}{2}$$

putting $y' = z$ we obtain $y'' = z'$ and the given equation becomes $z' = 2x + \frac{z}{2}$

Now, $z_0' = 2(1) + \frac{2}{2} = 3$

$$z_1' = 2(1.1) + \frac{2.3178}{2} = 3.3589$$

$$z_2' = 2(1.2) + \frac{2.6725}{2} = 3.73625$$

$$z_3' = 2(1.3) + \frac{3.0657}{2} = 4.13285$$

We have the following table.

x	$x_0 = 1$	$x_1 = 1.1$	$x_2 = 1.2$	$x_3 = 1.3$
y	$y_0 = 2$	$y_1 = 2.2156$	$y_2 = 2.4649$	$y_3 = 2.7514$
$y' = z$	$z_0 = 2$	$z_1 = 2.3178$	$z_2 = 2.6725$	$z_3 = 3.0657$
$y'' = z'$	$z_0' = 3$	$z_1' = 3.3589$	$z_2' = 3.73625$	$z_3' = 4.13285$

we first consider Milne's predictor formulae,

$$y_4^{(P)} = y_0 + \frac{4h}{3}\left(2z_1 - z_2 + 2z_3\right)$$

$$z_4^{(P)} = z_0 + \frac{4h}{3}\left(2z_1' - z_2' + 2z_3'\right)$$

On substituting the appropriate values from the table we obtain

$$y_4^{(P)} = 3.0793 \quad \text{and} \quad z_4^{(P)} = 3.4996$$

Next we consider Milne's corrector formulae,

$$y_4^{(C)} = y_2 + \frac{h}{3}\left(z_2 + 4z_3 + z_4\right)$$

$$z_4^{(C)} = z_2 + \frac{h}{3}\left(z_2' + 4z_3' + z_4'\right)$$

We have, $z_4' = 2x_4 + \frac{z_4^{(P)}}{2} = 2(1.4) + \frac{3.4996}{2} = 4.5498$

Hence by substituting the appropriate values in the corrector formulae we obtain

$$y_4^{(C)} = 3.0794 \quad \text{and} \quad z_4^{(C)} = 3.4997$$

Thus the required value of y is 3.0794 at $x = 1.4$.

4 a)

$$w = z + \frac{1}{z}$$

$$u + iv = re^{i\theta} + \frac{1}{r}e^{-i\theta}$$

$$= r[\cos\theta + i\sin\theta] + \frac{1}{r}(\cos\theta - i\sin\theta)$$

$$\Rightarrow u = \left(r + \frac{1}{r}\right)\cos\theta \quad -(1), \quad v = \left(r - \frac{1}{r}\right)\sin\theta \quad -(2)$$

Case (1) :- $|z| = 1$ in Z-plane

$\Rightarrow r = 1$.

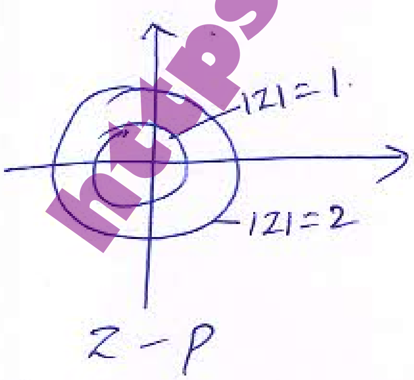
$u = 2\cos\theta$, $v = 0$.

is u-axis in w-plane with $(-2, 2)$

Case (2) :- $|z| = 2$ in Z plane.

$$\frac{u^2}{(r + 1/r)^2} + \frac{v^2}{(r - 1/r)^2} = 1.$$

$r = 2$, $$\frac{u^2}{(5/2)^2} + \frac{v^2}{(3/2)^2} = 1.$$



The $\circ^{le}$ $|z| = 1$ maps onto u-axis b/w -2 to 2 on ~~w-plane~~

& $|z| = 2$ maps onto an ellipse whose centre $(0,0)$ in w-plane.

b) Find the bilinear transformation that transforms the points $0, i, \infty$ onto the points $1, -i, -1$ respectively.

Sol.

Let $w = \dfrac{az+b}{cz+d}$ be the required bilinear transformation.

$z = \infty$, $w = -1$; the bilinear transformation is to be written in the form,

$$w = \frac{z[a+b/z]}{z[c+d/z]} = \frac{a+(b/z)}{c+(d/z)}$$

$$\therefore \quad -1 = \frac{a+0}{c+0} \quad (\because 1/z = 0 \text{ when } z = \infty)$$

i.e., $a + c = 0$ .... (1)

$z = i$, $w = -i$; $-i = \dfrac{ai+b}{ci+d}$

ie., $ai + b - c + di = 0$ ..... (2)

$z = 0$, $w = 1$; $1 = \dfrac{0+b}{0+d}$

i.e., $b - d = 0$ .... (3)

Now (1) + (2) gives

$(1+i)a + b + id = 0$ ... (4)

Let us solve (3) and (4) by writing them in the form

$0a + 1b - 1d = 0$ ... (3)

$(1+i)a + 1b + id = 0$ ... (4)

Applying the cross multiplication we have.

$$\frac{a}{\begin{vmatrix} 1 & -1 \\ 1 & i \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 0 & -1 \\ 1+i & i \end{vmatrix}} = \frac{d}{\begin{vmatrix} 0 & 1 \\ 1+i & 1 \end{vmatrix}}$$

$$\frac{a}{i+1} = \frac{-b}{1+i} = \frac{d}{-(1+i)} \quad \text{or} \quad \frac{a}{1} = \frac{b}{-1} = \frac{d}{-1}$$

$$a = 1, \ b = -1, \ d = -1$$

Also from (1) $c = -a \quad \therefore c = -1$

Substituting the values of a, b, c, d the assumed bilinear transformation becomes

$$w = \frac{1 \cdot z - 1}{-1 \cdot z - 1} = \frac{\cancel{-}(1-z)}{\cancel{-}(1+z)}$$

Thus $w = \frac{1-z}{1+z}$ is the required bilinear transformation.

Further, the invarients points are obtained by takin $w = z$

i.e., $z = \frac{1-z}{1+z}$ or $z + z^2 = 1 - z$

i.e., $z^2 + 2z - 1 = 0$

$$\therefore z = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

Thus the invarient points are $-1+\sqrt{2}$ and $-1-\sqrt{2}$.

c) State and prove Cauchy's integral formula and hence generalized Cauchy's integral formula.

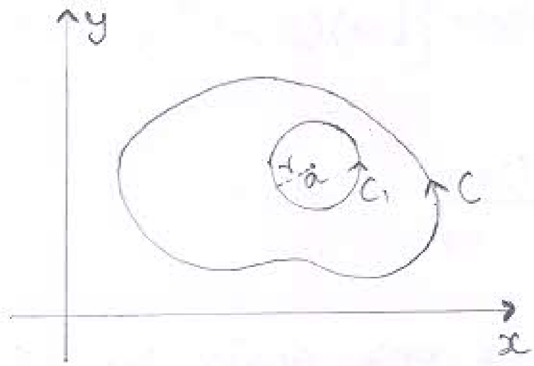
Sol.

If $f(z)$ is analytic inside and on a simple closed curve C and if 'a' is any point within C then

$$f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z-a} \, dz$$

Proof: Since 'a' is a point within c, we shall enclose it by a circle C_1 with $z = a$ as centre and r as radius such that C_1 lies entirely within C.

The function $\frac{f(z)}{z-a}$ is analytic inside and on the boundary of the annular region between C and C_1



Now as a consequence of Cauchy's theorem,

$$\int_C \frac{f(z)}{z-a}\,dz = \int_{C_1} \frac{f(z)}{z-a}\,dz \quad \ldots\ldots (1)$$

The equation of C_1 (circle with centre 'a' and radius 'r') can be written in the form $|z-a| = r$. That is equivalent to,

$$z-a = re^{i\theta} \text{ or } z = a + re^{i\theta},\ 0 \le \theta \le 2\pi \quad dz = ire^{i\theta}\,d\theta.$$

Using these results in the RHS of (1) we have,

$$\int_C \frac{f(z)}{z-a}\,dz = \int_{\theta=0}^{2\pi} \frac{f(a+re^{i\theta})}{re^{i\theta}}\, ire^{i\theta}\,d\theta$$

i.e. $$\int_C \frac{f(z)}{z-a}\,dz = i \int_{\theta=0}^{2\pi} f(a+re^{i\theta})\,d\theta$$

This is true for any $r > 0$ however small. Hence as $r \to 0$ we get,

$$\int_C \frac{f(z)}{z-a}\,dz = i \int_{\theta=0}^{2\pi} f(a)\,d\theta = if(a)\,[\theta]_0^{2\pi} = 2\pi i f(a)$$

Thus $$f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z-a}\,dz \quad \text{[Cauchy's integral formula]}$$

Applying Leibnitz rule for differentiation under the integral sign we have,

$$f'(a) = \frac{1}{2\pi i} \int_C f(z) \frac{\partial}{\partial a}\left[\frac{1}{z-a}\right] dz$$

i.e., $f'(a) = \frac{1}{2\pi i} \int_C f(z) \cdot \{(-1)(z-a)^{-2} \cdot (-1)\} \, dz$

i.e., $f'(a) = \frac{1!}{2\pi i} \int_C \frac{f(z)}{(z-a)^2}$

Applying Leibnitz rule once again for (2) we obtain

$$f''(a) = \frac{1!}{2\pi i} \int_C f(z) \frac{\partial}{\partial a} \left[(z-a)^{-2} \right] dz$$

$$= \frac{1!}{2\pi i} \int_C f(z) \cdot (-2)(z-a)^{-3} (-1) \, dz$$

i.e., $f''(a) = \frac{2!}{2\pi i} \int_C \frac{f(z)}{(z-a)^3} \, dz$

Continuing like this, after differentiating n times we obtain,

$$f^{(n)}(a) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z-a)^{n+1}} \, dz$$

Here $f^{(n)}(a)$ denotes the n^{th} deriviate of $f(z)$ at $z = a$.

3a. Analytic function: A function $f(z)$ which is single valued & possesses a unique derivative w.r.to z at all points of a region R.

$f(z) = u + iv$

$|f(z)| = \sqrt{u^2+v^2}$. Let $\sqrt{u^2+v^2} = c$.

$\Rightarrow u^2+v^2 = c^2$ — (1)

diff (1) p.w.r.to 'x'

$2uu_x + 2vv_x = 0$. $\Rightarrow uu_x + vv_x = 0$. — (2)

diff (1) p.w.r.to y

$2uu_y + 2vv_y = 0 \Rightarrow uu_y + vv_y = 0$ — (3)

(3) becomes $-uu_x + vu_x$. — (4)

(By C-R eqs)

Squaring & adding (2) & (4)

$(u^2+v^2)[u_x^2 + v_x^2] = 0$.

$\Rightarrow u_x^2 + v_x^2 = 0$ as $u^2+v^2 = c^2$.

$\Rightarrow |f'(z)|^2 = u_x^2 + v_x^2 = 0$.

$\Rightarrow |f'(z)| = 0$.

$\Rightarrow f'(z) = 0$.

$\Rightarrow f(z) = K$

= constant.

$\therefore$ An analytic func with constant modulus is constant.

b. $u = x^2 - y^2$, $\quad v = \dfrac{y}{x^2+y^2}$

$u_x = 2x$. $\quad v_x = -\dfrac{2xy}{(x^2+y^2)^2}$

$u_y = -2y$

$u_{xx} = 2$ $\quad v_y = \dfrac{x^2-y^2}{(x^2+y^2)^2}$

$u_{yy} = -2$

$v_{xx} = \dfrac{2y(3x^2-y^2)}{(x^2+y^2)^3}$

$v_{yy} = \dfrac{2y(y^2-3x^2)}{(x^2+y^2)^3}$.

$u_{xx} + u_{yy} = 0$.

$v_{xx} + v_{yy} = 0$

$\Rightarrow$ u & v are harmonic functions.

$u_x = 2x$, $\quad u_y = -2y$

By C-R eqs

$u_x = v_y$ & $v_x = -u_y$

= constant.

$\frac{\partial v}{\partial y} = 2x$, $\frac{\partial v}{\partial x} = 2y$.

on int on int.

$v = 2xy + f(x)$ $v = 2xy + g(y)$

on comparing

$v = 2xy$, $f(x) = 0$, $g(y) = 0$.

From Given $v = \frac{y}{x^2+y^2}$

∴ u & v are harmonic functions but are not harmonic conjugates.

C. $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ & $f(\pi/2) = 0$.

$$u_x - v_x = \frac{\{(-\sin x + \cos x)(2\cos x - \cosh y)\} - (\cos x + \sin x - e^{-y}) - 2\sin x}{[2(\cos x - \cosh y)]^2} \quad -(1)$$

$$u_y - v_y = -\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial y}\right)$$

$$= \frac{-e^{-y}[2(\cos x - \cosh y)] - (\cos x + \sin x - e^{-y})(-2\sinh y)}{2(\cos x - \cosh y)^2} \quad -(2)$$

Solving ① & ②

$$u_x = \frac{1}{4(\cos x - \cosh y)^2}\left[\{(\sin x - \cos x)\cosh y + 1 - e^{-y}\sin x\} - \{e^{-y}(\cos x - \cosh y) + (\cos x + \sin x - e^{-y})\sinh y\}\right]$$

$$= \phi_1(x, y)$$

$$u_y = \frac{1}{4(\cos x - \cosh y)^2}\left[\{(\sin x - \cos x)\cosh y + 1 - e^{-y}\sin x\} + \{e^{-y}(\cos x - \cosh y) + (\cos x + \sin x - e^{-y})\sin y\}\right]$$

$$\phi_1(z, 0) = \frac{1}{2(1-\cos z)}, \quad \phi_2(z,0) = 0.$$

$$f'(z) = u_x - i u_y = \phi_1(z,0) - i\phi_2(z,0)$$

$$= \frac{1}{2 \cdot 2\sin^2 z/2} = \frac{1}{4}\csc^2 z/2$$

on int. $f(z) = -1/2 \cot z/2 + C$

$$f(\pi/2) = 0 \Rightarrow C = 1/2$$

$$\therefore f(z) = 1/2(1 - \cot z/2)$$

Part B

6. a. Let A be the event in which sum of the nos is 4. Let B be the event in which no on both dice is even.

Favourable cases for $A = 3$ $\quad P(A) = 3/36$

ie $(1,3), (2,2), (3,1)$. $\quad P(A\cap B) = 1/36$

Favourable cases for $B = 1$ ie $(2,2)$

$$\therefore P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{1/36}{3/36} = 1/3$$

PART-B

6)

a) Total no. of outcomes in 1 dice = 6

$\therefore$ Total no. of outcomes when 2 dice are rolled $= 6 \times 6$

$= 36$

Favourable outcome = Sum of numbers obtained = 4, with both numbers on dice ~~to~~ is even

$\therefore$ ~~Only number~~

$\therefore$ Favourable Case = (2, 2)

$\therefore$ Prob of getting 2 on dice 1 $= \frac{1}{6}$

Prob of getting 2 in 2^{nd} dice $= \frac{1}{6}$

$\therefore$ Prob of getting 2 in both 1^{st} and 2^{nd} dice $= \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

b) Given

$P(\bar{A} \cap \bar{B}) = \frac{7}{12}$, $P(A \cap \bar{B}) = \frac{1}{6} = P(\bar{A} \cap B)$.

$P(\bar{A} \cap \bar{B}) = \frac{7}{12}$

$\Rightarrow P(\overline{A \cup B}) = \frac{7}{12}$

$1 - P(A \cup B) = \frac{7}{12}$

$\Rightarrow P(A \cup B) = 1 - \frac{7}{12} \Rightarrow P(A \cup B) = \frac{5}{12}$

$\bar{B}$ can be written as $\cancel{(A \cup \bar{A}) \cup \bar{B}}$ $(A \cup \bar{A}) \cap \bar{B}$

$= \cancel{S \cup P} = S \cap \bar{B}$

$= \bar{B}$

$\Rightarrow$ $\bar{B} = (A \cup \bar{A}) \cap \bar{B}$

$\bar{B} = (A \cap \bar{B}) \cup (\bar{A} \cap \bar{B})$

$\Rightarrow \cancel{\bar{B} = \frac{1}{6}}\ P(\bar{B}) = P[(A \cap \bar{B}) \cup (\bar{A} \cap \bar{B})]$

$\Rightarrow P(\bar{B}) = P(A \cap \bar{B}) + P(\bar{A} \cap \bar{B})$

$\Rightarrow P(\bar{B}) = \frac{1}{6} + \frac{7}{12}$

$= \frac{9}{12} = \frac{3}{4}$

$\therefore P(B) = 1 - P(\bar{B})$

$= 1 - \frac{3}{4} \Rightarrow P(B) = \frac{1}{4}$

Similarly

$\bar{A} = (B \cup \bar{B}) \cap \bar{A}$

$\Rightarrow \bar{A} = (B \cap \bar{A}) \cup (\bar{A} \cap \bar{B})$

$\Rightarrow P(\bar{A}) = P[(B \cap \bar{A}) \cup (\bar{A} \cap \bar{B})]$

$\Rightarrow P(\bar{A}) = P(B \cap \bar{A}) + P(\bar{A} \cap \bar{B})$

$\Rightarrow P(\bar{A}) = \frac{1}{6} + \frac{7}{12} \Rightarrow P(\bar{A}) = \frac{9}{12} = \frac{3}{4}$

$\Rightarrow P(A) = \cancel{\frac{1}{P}}\ 1 - \frac{3}{4} \Rightarrow P(A) = \frac{1}{4}$

For Mutually disjoint events

$P(A \cup B) = P(A) + P(B)$; $P(A \cup B) = \frac{5}{12}$

$P(A) + P(B) = \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2} \neq \frac{5}{12}$

$\therefore P(A \cup B) \neq P(A) + P(B)$

$\therefore$ The events are not mutually disjoint

For independent events.

$P(A \cap B) = P(A) \cdot P(B)$

$P(A) \cdot P(B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

$P(A \cap B) = P(A) - P(\bar{A} \cap B) = \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} \Rightarrow P(A \cap B) = \frac{1}{12}$

$\Rightarrow P(A\cap B) \neq P(A).P(B)$

$\Rightarrow$ ~~Both~~ The events are not independent

$\therefore$ Both the events A and B are neither mutually disjoint nor independent.

$P(A|B) + P(B|A)$

$$= \frac{P(A\cap B)}{P(B)} + \frac{P(A\cap B)}{P(A)}$$

$$= \frac{1/12}{1/4} + \frac{1/12}{1/4}$$

$$= \frac{4}{12} + \frac{4}{12} = \frac{8}{12} = \frac{2}{3}$$

$P(\bar{A}/\bar{B}) + P(\bar{B}/\bar{A})$

$$= \frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} + \frac{P(\bar{B}\cap\bar{A})}{P(\bar{A})}$$

$$= \frac{7/12}{3/4} + \frac{7/12}{3/4}$$

$$= \frac{7\times 4}{3\cancel{12}\times 3} + \frac{7\times 4}{3\cancel{12}\times 3}$$

$$= \frac{7}{9} + \frac{7}{9}$$

$$= \frac{14}{9}$$

c) Let A, B and C be the events of items produced by machines M_1, M_2, M_3 respectively

$\therefore$ Probability of item produced by Machine 1 $[P(A)] = 0.25$

Probability of item produced by Machine 2 $[P(B)] = 0.3$

Probability of item produced by Machine 3 $[P(C)] = 1-(0.25+0.3)$
$= 0.45$

Let D be the event of producing a defective item.

$\therefore$ Prob of defective item produced by Machine M1 $[P(D/A)] = 0.05$

Prob of defective item produced by machine M2 $[P(D/B)] = 0.04$

Prob of defective item produced by machine M1 $[P(D/C)] = 0.03$

Prob of finding faulty item produced by machine with highest output i.e Machine M3.

$$\therefore P(C/D) = \frac{P(C).\ P(D/C)}{P(A).P(D/A) + P(B).\ P(D/B) + P(C).P(D/C)}$$

$$= \frac{0.45 \times 0.03}{0.25 \times 0.05 + 0.3 \times 0.04 + 0.45 \times 0.03}$$

$$= 0.355$$

7)

a) ② Let p be the probability of guessing correct answer and q be the probability of guessing wrong answer.

$$\therefore p = \frac{1}{4}$$

$$q = \frac{3}{4}$$

Total no. of questions $(n) = 10$

① Probability of guessing atleast 6 answers correctly out of 10.

$p = 1/2,\ q = 1/2$

$P(x \geq 6) = 0.377$

Let x be the no. of times a correct answer is said.

$$\therefore P(x) = {}^{10}C_x\ p^x q^{n-x}$$

$\therefore$ Prob of getting more than 6 answers correct $= P(X \geq 6)$

$$= P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= 0.01622 + 3.0899 \times 10^{-3} + 3.8623 \times 10^{-4} + 2.8610 \times 10^{-5} + 9.5367 \times 10^{-7}$$

$$\Rightarrow P(X \geq 6) = 0.0197$$

4b)

b) The continous probability distribution having the probability density function $f(x)$ given by

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & \text{for } x > 0 \\ 0 & \text{otherwise, where } \alpha > 0 \end{cases}$$

is known as exponential distribution.

$$\text{Mean} = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{0} 0 + \int_{0}^{\infty} x \cdot \alpha e^{-\alpha x}\, dx$$

$$= \alpha \int_{0}^{\infty} x e^{-\alpha x}\, dx$$

$$= \alpha \left[x \frac{e^{-\alpha x}}{-\alpha} \Big]_0^{\infty} - 1 \frac{e^{-\alpha x}}{\alpha^2} \right]_0^{\infty}$$

$$= \alpha \left[(0-0) - 1\left(0 - \frac{1}{\alpha^2}\right) \right]$$

$$= \alpha \left[\frac{1}{\alpha^2} \right] \Rightarrow \text{Mean} = \mu = \frac{1}{\alpha}$$

$$\text{Varience}(\sigma^2) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

$$= \int_{-\infty}^{0} (x-\mu)^2 \cdot 0\, dx + \int_{0}^{\infty} (x-\mu)^2 \alpha e^{-\alpha x}\, dx$$

$$= \int_{0}^{\infty} (x-\mu)^2 \alpha e^{-\alpha x}\, dx$$

$$= \cancel{(x-\mu)^2} \alpha \left[(x-\mu)^2 \frac{e^{-\alpha x}}{-\alpha} \Big]_0^{\infty} - 2(x-\mu) \frac{e^{-\alpha x}}{\alpha^2} \Big]_0^{\infty} + 2 \frac{e^{-\alpha x}}{-\alpha^3} \right]_0^{\infty}$$

$$= \alpha \left[-\left(0 - \frac{\mu^2}{\alpha}\right) - 2\left[0 - \frac{-\mu}{\alpha^2}\right] \cancel{+2} - 2\left(0 - \frac{1}{\alpha^3}\right) \right]$$

But $\mu = \frac{1}{\alpha}$

$$= \alpha \left[\frac{1}{\alpha^3} - \frac{2}{\alpha^3} + \frac{2}{\alpha^3} \right] = \alpha \left[\frac{1}{\alpha^3} \right] = \frac{1}{\alpha^2}$$

$\therefore \sigma^2 = \cancel{\phi}\ \frac{1}{\alpha^2}$

$\Rightarrow \sigma = \frac{1}{\alpha}$

c)

$\mu = 30$

$\sigma = 5$

$z = \frac{X-\mu}{\sigma}$ —— ①

i) $26 \le X \le 40$

Put $X = 26$ in ①

$\Rightarrow z = \frac{-4}{5} = -0.8$

Put $X = 40$ in ①

$z = \frac{10}{5} = 2$

$\therefore P(-0.8 \le z \le 2)$

$= \cancel{\phi(2) - \phi(-0.8)} = \cancel{\phi(z<2) + \phi(z>-0.8)}$

$= \cancel{\phi(2) + \phi(0.8)}$

$= \phi(0 \le z \le 2) + \phi(-0.8 \le z \le 0)$

$= \phi(2) + \phi(0.8)$ $\quad [\because \phi(-.8 \le z \le 0) = \phi(0 \le z \le 0.8) = \phi(0.8)]$

$= 0.4772 + 0.2881$

$= 0.7653$

ii) $X \ge 45$

Put $X = 45$ in ①

$z = \frac{45-30}{5} \Rightarrow z = 3$

$\therefore P(z \ge 3)$

$= P(z \ge 0) - P(0 \le z \le 3)$

$= 0.5 - \phi(3)$

$= 0.5 - 0.4987$

$= 0.0013$

iii) $|X-30|>5$

$= X-30>5$ or $-X+30>5$

$\Rightarrow X>35$ or $-X>-25$

$\Rightarrow X>35$ or $X<25$

$= 25<X<35$

Put $X=25$ in ①

$z=\frac{25-30}{5}=-1$

Put $X=35$ in ①

$z=\frac{35-30}{5}=1$

$\therefore P(-1<z<1)$

$= \cancel{2\phi} = 2P(0<z<1)$

$= 2\times\phi(1)$

$= 2\times 0.3413$

$= 0.6826$

$P[|x-30|>5] = 1-P$
$= 1-0.6826$
$= 0.3174$.

8) a) Given

$\mu = 800$ hrs

$\sigma = 60$ hrs

$z=\frac{X-\mu}{\sigma} \Rightarrow z=\frac{X-800}{60}$ — ①

i) between 790 hrs & 810 hrs

Put $X=790$ in ①

$z_1=\frac{790-800}{60}=-0.167$

Put $X=810$ in ①

$z_2=\frac{810-800}{60}=+0.167$

$\therefore P(-0.167<z<0.167)$

$= 2P(0<z<0.167)$

b) 5 coins, tossed 320 times.

b.

No. of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Given $\psi^2_{0.05}(5) = 11.07$

→ The data gives the observed frequency & we need to calculate the expected frequencies.

∴ Prob of getting heads is $= \frac{1}{2} = p$

$\therefore q = 1 - p = \frac{1}{2}$

The binomial distribution fit is, $N(p+q)^n$

$= 320\left(\frac{1}{2}+\frac{1}{2}\right)^5$

The theoretical frequencies of getting 0, 1, 2, 3, 4 or 5 success with 5 coins are respectively the successive terms of the binomial expansion.

They are respectively $320 \times \frac{1}{2^5}, 320 \times {}^5C_1 \times \frac{1}{2^5}, 320 \times {}^5C_2 \times \frac{1}{2^5},$

$320 \times {}^5C_3 \times \frac{1}{2^5}, 320 \times {}^5C_4 \times \frac{1}{2^5}, 320 \times {}^5C_5 \times \frac{1}{2^5}$

$= 10, 50, 100, 100, 50, 10$

We have table of observed & expected frequency

O_i	6	27	72	112	71	32
E_i	10	50	100	100	50	10

$$\psi^2 = \sum \left\{\frac{(O_i - E_i)^2}{E_i}\right\}$$

$$= \frac{16}{10} + \frac{529}{50} + \frac{784}{100} + \frac{144}{100} + \frac{441}{50} + \frac{484}{10}$$

$\Rightarrow \psi^2 = 78.68 > \psi^2_{0.05} = 11.07$

∴ The hypothesis that the data follows a binomial distribution is rejected

a) Given

$\mu = 800, \quad \sigma = 60, \quad n = 16$

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 60/4$
$= 15$

We have $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{\bar{x} - 800}{15}$ — ①

To find $P(790 < \bar{x} < 810)$

If $\bar{x} = 790, \; z = -0.67$

If $\bar{x} = 810, \; z = 0.67$

$\therefore P(-0.67 < z < 0.67) = 2P(0 < z < 0.67)$
$= 2 \times \phi(0.67)$
$= 2 \times 0.2486$
$= 0.4972$

$\therefore P(790 < \bar{x} < 810) = 0.4972$

b) To find $P(\bar{x} < 785)$

Put $\bar{x} = 785$ in ①

$z = \dfrac{785 - 800}{15} \Rightarrow z = -1$

$\Rightarrow$

$P(z < -1) = P(z > 1)$
$= 0.5 - \phi(1)$
$= 0.5 - 0.3413$
$= 0.1587$

$\therefore P(\bar{x} < 785) = 0.1587$

c) To find $(\bar{x} > 820)$

If $\bar{x} = 820 \Rightarrow z = 1.33$ from ①

$\Rightarrow P(z > 1.33) = P(z > 0) - P(0 < z < 1.33)$
$= 0.5 - \phi(1.33)$
$= 0.5 - 0.4082$
$= 0.0918$

$\therefore P(\bar{x} > 820) = 0.0918$

c)

Let p_1 and p_2 be the proportion of smokers ~~in~~ among students and lecturers resp.

$p_1 = \frac{2}{60} = 0.033$; $p_2 = \cancel{\frac{5}{16}}\ \frac{5}{17} = 0.2941$

Let H_0 be the null proportion that there is no significant difference between the students & lecturers in smoking

$$p = \frac{n_1p_1 + n_2p_2}{n_1+n_2} = \frac{60 \times 0.033 + 17 \times \cancel{0.0}\ 0.2941}{60+17}$$

$$= \frac{2+5}{77} = \frac{7}{77} = \frac{1}{11} \Rightarrow p = 0.0909$$

$q = 1-p$
$= 0.909$

Consider $z = \frac{p_2 - p_1}{\sqrt{pq(1/n_1 + 1/n_2)}}$

$$= \frac{0.2941 - 0.033}{\sqrt{(0.0909)(0.909)\left(\frac{1}{60} + \frac{1}{17}\right)}}$$

$= 3.301$

$z = 3.301$ $\begin{cases} > z_{.05} = 1.96 \text{ (two tailed test)} \\ > z_{.01} = 2.58 \text{ (two tailed test)} \end{cases}$

Thus null hypothesis is rejected both at 5% and 1% levels of significance.

5.

a. $x^2y'' + xy' + (x^2 - 1/4)\,y = 0.$

This is of the form

$x^2y'' + xy' + (x^2 - n^2)\,y = 0$

on comparing

$n = \pm 1/2$

Solution in terms of Bessel's is given in the form.

$y = a\,J_{1/2}(x) + b\,J_{-1/2}(x).$

$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\,\sin x. \quad , J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x.$

$y = a\sqrt{\frac{2}{\pi x}}\sin x + b\sqrt{\frac{2}{\pi x}}\cos x$

$\therefore y = c_1 \frac{\sin x}{\sqrt{x}} + c_2 \frac{\cos x}{\sqrt{x}}$

b. $(1-x^2)y'' - 2xy' + n(n+1)y = 0 \quad -①$

The coeff of $y'' = 1 - x^2$
$= P_0(x). \neq 0$ at $x = 0.$

Let $y = \sum_{r=0}^{\infty} a_r x^r \quad -②$ be the series solution of ①

$$y' = \sum_0^\infty a_r\, r\, x^{r-1}, \quad y'' = \sum_0^\infty a_r\, r(r-1) x^{r-2}$$

① becomes.

$$\sum_0^\infty a_r\, r(r-1)x^{r-2} - \sum_0^\infty a_r r(r-1)x^r - \sum_0^\infty 2a_r r x^r$$
$$+ n(n+1)\sum_0^\infty a_r x^r = 0.$$

coeff of x^{-2} $\Rightarrow a_0 (0)(-1) = 0 \Rightarrow a_0 \neq 0$.

coeff of x^{-1} $\Rightarrow a_1 (1)(0) = 0 \Rightarrow a_1 \neq 0$.

equate coeff of x^r

$$a_{r+2} = -\frac{[n(n+1) - r^2 - r]}{(r+2)(r+1)} a_r$$

putting $r = 0, 1, 2, 3 - -$

$$a_2 = -\frac{n(n+1)}{2} a_0, \quad a_3 = -\frac{(n^2+n-2)}{6} a_1$$
$$= -\frac{(n-1)(n+2)}{6} a_1$$

$$a_4 = -\frac{(n-2)(n+3)}{12} \cdot -\frac{n(n+1)}{2} a_0.$$

Sub all the values in expanded form of ②

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + - - -$$

$$y = (a_0 + a_2 x + a_4 x^2) + (a_1 x + a_3 x^3 + a_5 x^5 + \ldots)$$

$$y = a_0\left[1 - \frac{n(n+1)}{2!} x^2 + \frac{n(n+1)(n-2)(n+3)}{4!} x^4 - - -\right]$$
$$+ a_1\left[x - \frac{(n-1)(n+2)}{3!} x^3 + \frac{(n-1)(n+2)(n-3)(n+4)}{5!} x^5 - -\right]$$

$\therefore y = a_0 u(x) + a_1 v(x)$. This is the series sol c.

Rodrigue's formula

$$P_n(x) = \frac{1}{2^n \cdot n!} \frac{d^n}{dx^n}\left[(x^2-1)^n\right]$$

By putting $n=4$

$$P_4(x) = \frac{1}{8}\left[35x^4 - 30x^2 + 3\right]$$

$$P_4(1) = \frac{1}{8}\left[35-30+3\right] = 1$$

The property $P_n(1)=1$ in respect of Legendre polynomials is satisfied for $P_4(x)$.

Also $\int_{-1}^{1} x^3 P_4(x)\,dx = \int_{-1}^{1} x^3 \cdot \frac{1}{8}\left[35x^4 - 30x^2 + 3\right]dx$.

$$= \frac{1}{8}\int_{-1}^{1}\left[35x^7 - 30x^5 + 3x^3\right]dx.$$

$$= \frac{1}{8}\left\{35\left[\frac{x^8}{8}\right]_{-1}^{1} - 30\left[\frac{x^6}{6}\right]_{-1}^{1} + 3\left[\frac{x^4}{4}\right]_{-1}^{1}\right\}$$

$$= \frac{1}{8}\left\{\frac{35}{8}(1-1) - \frac{30}{6}(1-1) + \frac{3}{4}(1-1)\right\}$$

$$= 0.$$