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Model Question Paper-I with effect from 2016-17

USN | | | | | | | | | | | **15MAT41**

Fourth Semester B.E.(CBCS) Examination
Engineering Mathematics-IV
(Common to all Branches)

Time: 3 Hrs   Max.Marks: 80

Note: Answer any FIVE full questions, choosing at least ONE question from each module. Statistical tables may be provided.

Module-I

1. (a) Using Taylor's series method, solve the initial value problem $\frac{dy}{dx} = xy^2 - 1, y(0) = 1$ and hence find the the value of y at the point $x = 0.1$. **(05 Marks)**

(b) Employ fourth order Runge - Kutta method to solve $\frac{dy}{dx} = (y^2 - x^2)/(y^2 + x^2), y(0) = 1$, at $x = 0.2$. (Take $h = 0.2$) **(05 Marks)**

(c) Using Adam-Bashforth predictor-corrector method to solve $\frac{dy}{dx} = x^2(1 + y)$ given that $y(1) = 1, y(1.1) = 1.2330, y(1.2) = 1.5480, \& \ y(1.3) = 1.9790$ to find $y(1.4)$. **(06 Marks)**

OR

2. (a) Using modified Euler's method find $y(0.1)$, given $\frac{dy}{dx} + y - x^2 = 0$ with $y(0) = 1$. Perform two iterations at each step, taking $h = 0.05$. **(05 Marks)**

(b) Use fourth order Runge - Kutta method to find $y(1.1)$, given $\frac{dy}{dx} = xy^{1/3}, y(1) = 1$. (Take $h = 0.1$) **(05 Marks)**

(c) Apply Milne's predictor-corrector formulae to compute $y(0.8)$, given **(06 Marks)**
$\frac{dy}{dx} = x - y^2$ and

x	0	0.2	0.4	0.6
y	0	0.0288	0.0788	0.1799

Module-II

3. (a) Given $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1, y(0) = 1, y'(0) = 0$, evaluate $y(0.1)$ using Runge - Kutta method. **(05 Marks)**

(b) Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. **(05 Marks)**

(c) Solve the Bessel's differential equation viz., $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ to write the complete solution in terms of $J_n(x)$. **(06 Marks)**

OR

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4. (a) Apply Milne's method to compute $y(0.8)$ given that $\frac{d^2y}{dx^2}=1-2y\frac{dy}{dx}$ and the following table of initial values:

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

(05 Marks)

(b) With usual notation, prove that $J_{1/2}(x)=\sqrt{(2/\pi x)}\sin x$.

(05 Marks)

(c) c) If $x^3+2x^2-x+1=aP_0(x)+bP_1(x)+cP_2(x)+dP_3(x)$, find the values of a,b,c,d

(06 Marks)

Module-III

5. (a) Derive Cauchy-Riemann equation in polar form.

(05 Marks)

(b) Using Cauchy's residue theorem to evaluate the integral $\int_c \frac{z^2dz}{(z-1)^2(z+2)}$ where C is the circle $|z|=3$

(05 Marks)

(c) Find the bilinear transformation that transforms the points $z_1=i, z_2=1, z_3=-1$ on to the points $w_1=1, w_2=0, w_3=\infty$ respectively.

(06 Marks)

OR

6. (a) State and prove Cauchy's integral formula.

(05 Marks)

(b) Given $u-v=(x-y)(x^2+4xy+y^2)$, find the analytic function $f(z)=u+iv$

(05 Marks)

(c) Discuss the transformation $w=z+(1/z), z\neq 0$.

(06 Marks)

Module-IV

7. (a) Derive mean and standard deviation of the binomial distribution.

(05 Marks)

(b) In a certain factory turning out razor blades, there is a small chance of 0.002 for a blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing i) no defective ii) one defective iii) two defective blades, in a consignment of 10,000 packets.

(05 Marks)

(c) The joint probability distribution for two random variables X and Y is given below:

X \ Y	-2	-1	4	5
1	0.1	0.2	0.0	0.3
2	0.2	0.1	0.1	0.0

Determine (i) marginal distribution of X and Y (ii) covariance of X and Y (iii) correlation between X and Y

(06 Marks)

OR

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8.(a) The average daily turn out in a medical store is Rs. 10,000 and the net profit is 8%. If the turn out has an exponential distribution, find the probability that the net profit will exceed Rs. 3000 each on two consecutive days.

(05 Marks)

(b) The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 and 75, given $\Phi(1) = 0.3413$.

(05 Marks)

(c) The Joint distribution of two random variables X and Y is as follows:

X \ Y	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Compute (i) $E(X)$ and $E(Y)$ (ii) $E(XY)$ (iii) $Cov(X, Y)$ &
(iv) $\rho(X,Y)$

(06 Marks)

Module-V

9. (a) Explain the terms:(i)Null hypothesis (ii)Confidence intervals (iii)Type I and Type II errors **(05marks)**

(b) A certain stimulus administered to each of the 12 patients resulted in the following change in the blood pressure 5,3,8,-1,3,0,6,-2,1,5,0,4. Can it be concluded that the stimulus will increase the blood pressure ($t_{0.05}$ for 11 d.f is 2.201).

(05 marks)

(c) Show that the Markov chain whose transition probability matrix $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreducible. Also, find the corresponding stationary probability vector. **(06 marks)**

OR

10. (a) In an elementary school examination the mean grade of 32 boys was 72 with a standard deviation of 8 while the mean grade of 36 girls was 75 with a standard deviation of 6. Test the hypothesis that the performance of girls is better than boys.

(05 marks)

(b) Four coins are tossed 100 times and the following results were obtained.

Number of Heads	0	1	2	3	4
Frequency	5	29	36	25	5

Fit a binomial distribution for the data and test the goodness of fit ($\chi^2_{0.05} = 9.49$ for 4 d.f.). **(05marks)**

(c) Every year, a man trades for his car for a new car. If he has Maruti, he trade it for a Ford. If he has a Ford, he trade it for a Hyundai. However, if he has a Hyundai, he is just as likely to trade it for a new Hyundai as to trade it for a Maruti or a Ford. In 2014 he bought his first car which was a Hyundai. Find the probability that he has (a) 2016 Ford (b) 2016 Hyundai (c) 2016 Maruti.

(06 marks)

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Model Question Paper-1 with effect from 2016-17

Fourth Semester B.E (CBCS) Examination

Engineering Mathematics - IV

(Common to all Branches)

15MAT41

1) a) using Taylor's series method, solve the IVP $\frac{dy}{dx} = xy^2 - 1$, $y(0)=1$ and hence find the value of y at the point $x=0.1$

By data $y' = \frac{dy}{dx} = xy^2 - 1$, $x_0 = 0$, $y_0 = 1$

The Taylor's series method

$$y(x) = y(x_0) + (x-x_0) y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \frac{(x-x_0)^3}{3!} y'''(x_0) + \cdots$$

the point $x_0 = 0$

$$\therefore y(x) = y(0) + x y'(0) + \frac{x^2}{2!} y''(0) + \frac{x^3}{3!} y'''(0) + \cdots \quad (1)$$

$y' = xy^2 - 1 \qquad \therefore y'(0) = -1$

$y'' = 2xyy' + y^2 \qquad y''(0) = 1$

$y''' = 2[xyy'' + yy' + x(y')^2] + 2yy' \qquad y'''(0) = -4$

(1) $\Rightarrow$

$$y(x) = 1 + x(-1) + \frac{x^2}{2}(1) + \frac{x^3}{6}(-4)$$

$$y(x) = 1 - x + \frac{x^2}{2} - \frac{2x^3}{3}$$

$$\boxed{y(0.1) = 0.9043}$$

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1) b) Employ fourth order Runge-kutta method to solve $\frac{dy}{dx} = \frac{(y^2-x^2)}{(y^2+x^2)}$, $y(0)=1$, at $x=0.2$, $h=0.2$

>> we have

$$f(x,y) = \frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}; \quad x_0=0, \ y_0=1, \ h=0.2$$

$$k_1 = hf(x_0, y_0) = 0.2 \, f(0,1) = 0.2(1) = 0.2$$

$$k_2 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right) = 0.2 \, f(0.1, 1.1) = 0.1967$$

$$k_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right) = 0.2 \, f(0.1, 1.0984) = 0.1967$$

$$k_4 = hf(x_0+h, y_0+k_3) = 0.2 \, f(0.2, 1.1967) = 0.1891$$

we have $y(x_0+h) = y_0 + \frac{1}{6}(k_1+2k_2+2k_3+k_4)$

$$\boxed{y(0.2) = 1.196}$$

1) c) using Adam-Bashforth predictor-Corrector method to solve $dy/dx = x^2(1+y)$ given that $y(1)=1$, $y(1.1)=1.2330$, $y(1.2)=1.5480$, $y(1.3)=1.9790$ to find $y(1.4)=?$

here $h=0.1$ difference

x	y	$y' = x^2(1+y)$
$x_0 = 1$	$y_0 = 1$	$y_0' = x_0^2(1+y_0) = 2$
$x_1 = 1.1$	$y_1 = 1.2330$	$y_1' = x_1^2(1+y_1) = 2.702$
$x_2 = 1.2$	$y_2 = 1.5480$	$y_2' = x_2^2(1+y_2) = 3.669$
$x_3 = 1.3$	$y_3 = 1.9790$	$y_3' = x_3^2(1+y_3) = 5.035$

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$$y_4^{(p)} = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$y_4^{(p)} = 2.573$$

Now $y_4' = f(x_4, y_4) = x_4^2(1+y_4) = 7.004$

$$y_4^{(c)} = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]$$

$y_4^{(c)} = 2.575$ twice

Hence $\boxed{y(1.4) = 2.575}$

(OR)

2) a) Using modified Euler's method find $y(0.1)$ given $\frac{dy}{dx} + y - x^2 = 0$ with $y(0) = 1$, perform two iterations at each step, taking $h = 0.05$.

>> I Stage By data

$x_0 = 0,\ y_0 = 1,\ f(x, y) = x^2 - y,\quad h = 0.05$

$f(x_0, y_0) = x_0^2 - y_0 = 0^2 - 1 = -1$

$x_1 = x_0 + h = 0.05 \quad \therefore \boxed{f(x_0, y_0) = -1}\ \boxed{x_1 = 0.05}$

$y(x_1) = y_1 \Rightarrow \boxed{y(0.05) = ?}$

Euler's formulae

$$y_1^{(0)} = y_0 + hf(x_0, y_0)$$

$$y_1^{(0)} = 1 + 0.05(-1)$$

$$\boxed{y_1^{(0)} = 0.95}$$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right] = 0.9513$$

$$\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right] = 0.9513$$

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II Stage $y(0.05) = 0.9513$

$x_0 = 0.05, \quad y_0 = 0.9513, \quad h = 0.05 \quad f(x_0, y_0) = x^2 - y$

$\therefore f(x_0, y_0) = -0.9488, \quad x_1 = x_0 + h = 0.1$

$y(x_1) = y_1 \Rightarrow \boxed{y(0.1) = ?}$

$y_1^{(0)} = y_0 + h f(x_0, y_0) = 0.9039$

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right] = 0.9052$

$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right] = 0.9052$

$\therefore \boxed{y(0.1) = 0.9052}$

2) b) Use fourth order Runge-kutta method to find $y(1.1)$, given $dy/dx = xy^{1/3}$, $y(1) = 1$, (take $h = 0.1$)

$\Rightarrow$ By data $f(x, y) = xy^{1/3}$, $x_0 = 1$, $y_0 = 1$

$x = x_0 + h \Rightarrow 1.1 - 1 = h \Rightarrow \boxed{h = 0.1}$

$k_1 = h f(x_0, y_0) = 0.1 f(1, 1) = 0.1 [xy^{1/3}] = 0.1$

$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= 0.1 \ f(1.05, 1.05) = 0.1 \ [xy^{1/3}] = 0.1067$

$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.1 f(1.05, 1.05335) = 0.1068$

$k_4 = h f(x_0 + h, y_0 + k_3)$

$= 0.1 \ f(1.1, 1.1068) = 1.01138$

$y(x_0 + h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$\boxed{y(1.1) = 1.1068}$

2) c) Apply Milne's predictor - Corrector formulae to Compute $y(0.8)$ given $dy/dx = x - y^2$ and

x	0	0.2	0.4	0.6
y	0	0.0288	0.0788	0.1799

> $h = 0.2$ by difference

x	y	$y' = x - y^2$
$x_0 = 0$	$y_0 = 0$	$y_0' = x_0 - y_0^2 = 0$
$x_1 = 0.2$	$y_1 = 0.0288$	$y_1' = x_1 - y_1^2 = 0.1992$
$x_2 = 0.4$	$y_2 = 0.0788$	$y_2' = x_2 - y_2^2 = 0.3938$
$x_3 = 0.6$	$y_3 = 0.1799$	$y_3' = x_3 - y_3^2 = 0.5676$
$x_4 = 0.8$	$y_4 = ?$	

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3') = 0.3039$$

$$y_4' = f(x_4, y_4) = x_4 - y_4^2 = 0.7076$$

$$y_4^{(C)} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4') = 0.2767$$

$$y_4' = f(x_4, y_4) = 0.7234$$

$$\boxed{y_4^{(C)} = 0.3046}$$

Module - II

3) a) Given $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$, $y(0) = 1$, $y'(0) = 0$ evaluate $y(0.1)$ using Runge-kutta method.

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Assume $\dfrac{dy}{dx} = z \Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{dz}{dx}$

$\therefore \dfrac{dz}{dx} - x^2 z - 2xy = 1$

Hence $\dfrac{dy}{dx} = z$, $\dfrac{dz}{dx} = 1 + 2xy + x^2 z$

$\therefore f(x,y,z) = z$ and $g(x,y,z) = 1 + 2xy + x^2 z$

$x_0 = 0$, $y_0 = 1$, $z_0 = 0$, and $h = 0.1$

$k_1 = h f(x_0, y_0, z_0) = 0$ $\qquad l_1 = h g(x_0, y_0, z_0) = 0.1$

$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$ $\qquad l_2 = h g\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$

$= 0.005$ $\qquad = 0.11$

$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$ $\qquad l_3 = h g\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$

$= 0.0055$ $\qquad = 0.11004$

$k_4 = h f(x_0 + h, y_0 + k_3, z_0 + l_3)$ $\qquad l_4 = h g(x_0 + h, y_0 + k_3, z_0 + l_3)$

$\qquad = 0.12022$

$k_4 = 0.011$

$y(x_0 + h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$\boxed{y(0.1) = 1.0053}$

3) b) Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials.

here $x = P_1(x)$, $1 = P_0(x)$, $x^2 = \frac{1}{3}P_0(x) + \frac{2}{3}P_2(x)$

$x^3 = \frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)$

$f(x) = \frac{2}{5}P_3 + \frac{3}{5}P_1 + 2\left[\frac{1}{3}P_0 + \frac{2}{3}P_2\right] - P_1 - 3P_0$

$\boxed{f(x) = \frac{2}{5}P_3(x) + \frac{4}{3}P_2(x) - \frac{2}{5}P_1(x) - \frac{7}{3}P_0(x)}$

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3) c) Bessel's differential Equation.

$$x^2y'' + xy' + (x^2 - n^2)y = 0 \quad --- (1)$$

coefficient of $y'' = x^2 = P_0(x)$ and $P_0(x) = 0$ at $x = 0$

Series sol^n is

$$y = \sum_{r=0}^{\infty} a_r x^{k+r} \longrightarrow (2)$$

$$y' = \sum_0^{\infty} a_r (k+r) x^{k+r-1} \qquad y'' = \sum_0^{\infty} a_r (k+r)(k+r-1) x^{k+r-2}$$

$(1) \Rightarrow$

$$\sum_0^{\infty} a_r x^{k+r} \left[(k+r)^2 - n^2\right] + \sum_0^{\infty} a_r x^{k+r+2} = 0$$

coefficent of x^k : $\quad a_0(k^2 - n^2) = 0$

$a_0 \neq 0, \quad k = \pm n.$

$---- \quad x^{k+1} : a_1\left[(k+1)^2 - n^2\right] = 0$

$a_1 = 0, \quad (k+1) = \pm n$ which can't be accepted as we have already $k = \pm n$.

$---- \quad x^{k+r} \quad (r \geq 2)$ to zero

$$a_r\left[(k+r)^2 - n^2\right] + a_{r-2} = 0$$

$$a_r = \frac{-a_{r-2}}{2nr + r^2} \qquad \text{where } k = +n.$$

$r = 2, 3, 4 \dots$

$a_1 = a_3 = a_5 = \dots = 0$

$$a_2 = \frac{-a_0}{4(n+1)} \qquad a_4 = \frac{a_0}{32(n+1)(n+2)} [-$$

$(2) \Rightarrow \quad y = x^k(a_0 + a_1x + a_2x^2 + \dots)$

sol^n for $k = +n, \quad y = y_1$

$$y_1 = x^n\left[a_0 - \frac{a_0}{4(n+1)} x^2 + \frac{a_0}{32(n+1)(n+2)} x^4 \dots\right]$$

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$$y_1 = a_0 x^n \left[1 - \frac{x^2}{2^2(n+1)} + \frac{x^4}{2^5(n+1)(n+2)} - \cdots \right] \quad (4)$$

also $k = -n$ be a sol^n denoted by y_2. eq^n (4)

$n \to -n$

$$y_2 = a_0 x^{-n} \left[1 - \frac{x^2}{2^2(-n+1)} + \frac{x^4}{2^5(-n+1)(-n+2)} - \cdots \right] \quad (5)$$

Complete sol^n of (1)

$y = A y_1 + B y_2$, where A, B arbitrary constants.

we shall now standardize the sol^n as in (4)

$$a_0 = \frac{1}{2^n \Gamma(n+1)} \quad \text{and } y_1 = Y_1$$

$$Y_1 = \left(\frac{x}{2}\right)^n \left[\frac{1}{\Gamma(n+1)} - \left(\frac{x}{2}\right)^2 \frac{1}{(n+1)\Gamma(n+1)} + \left(\frac{x}{2}\right)^4 \frac{1}{(n+1)(n+2)\Gamma(n+1).2} - \cdots \right]$$

$$Y_1 = \left(\frac{x}{2}\right)^n \left[\frac{1}{\Gamma(n+1)} - \left(\frac{x}{2}\right)^2 \cdot \frac{1}{\Gamma(n+2)} + \left(\frac{x}{2}\right)^4 \frac{1}{\Gamma(n+3).2} - \cdots \right]$$

$$= \left(\frac{x}{2}\right)^n \sum_{r=0}^{\infty} \frac{(-1)^r}{\Gamma(n+r+1) \cdot r!} \left(\frac{x}{2}\right)^{2r}$$

$$= \sum_{0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{\Gamma(n+r+1) \cdot r!}$$

This fu^n is called the Bessel function of the first kind of order n denoted by $J_n(x)$

Thus $$J_n(x) = \sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{\Gamma(n+r+1) \cdot r!}$$

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(OR)

4) a) Apply Milne's method to compute $y(0.8)$ given that $y'' = 1 - 2yy'$ and the foll^ng table of initial values.

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

$\Rightarrow y' = z \Rightarrow y'' = z'$

$\therefore$ The given d.eq^n $z' = 1 - 2yz$

x	$x_0 = 0$	$x_1 = 0.2$	$x_2 = 0.4$	$x_3 = 0.6$
y	$y_0 = 0$	$y_1 = 0.02$	$y_2 = 0.0795$	$y_3 = 0.1762$
$y' = z$	$z_0 = 0$	$z_1 = 0.1996$	$z_2 = 0.3937$	$z_3 = 0.5689$
$y'' = z'$	$z_0' = 1$	$z_1' = 0.992$	$z_2' = 0.9374$	$z_3' = 0.7995$

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2z_1 - z_2 + 2z_3) = 0.3049$$

$$z_4^{(P)} = z_0 + \frac{4h}{3}(2z_1' - z_2' + 2z_3') = 0.7055$$

$$y_4^{(c)} = y_2 + \frac{h}{3}(z_2 + 4z_3 + z_4) = 0.3045$$

$$z_4' = f(x_4, y_4, z_4) = 1 - 2y_4 z_4 = 0.5698$$

$$z_4^{(c)} = z_2 + \frac{h}{3}(z_2' + 4z_3' + z_4') = 0.7074$$

Applying the corrector formula again for y_4 we have.

$$y_4^{(c)} = 0.3046$$

Thus the required $\boxed{y(0.8) = 0.3046}$

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4) b) With usual notation PT $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

>> $J_n(x) = \sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{\Gamma(n+r+1)\cdot r!}$ —— (1)

$n = 1/2$

$$J_{1/2}(x) = \sum_{0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{1/2+2r} \frac{1}{\Gamma(3/2+r)\, r!}$$

$$= \sqrt{\frac{x}{2}} \sum_{0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{2r} \frac{1}{\Gamma(3/2+r)\, r!}$$

$$= \sqrt{\frac{x}{2}} \left[\frac{1}{\Gamma(3/2)} - \left(\frac{x}{2}\right)^2 \frac{1}{\Gamma(5/2)\, 1!} + \left(\frac{x}{2}\right)^4 \frac{1}{\Gamma(7/2)\cdot 2!} - \cdots \right] \text{—— (2)}$$

$$= \sqrt{\frac{x}{2}} \left[\frac{2}{\sqrt{\pi}} - \frac{x^2}{4}\cdot\frac{4}{3\sqrt{\pi}} + \frac{x^4}{16}\cdot\frac{8}{15\sqrt{\pi}\cdot 2} - \cdots \right]$$

$$= \sqrt{\frac{x}{2\pi}} \left[2 - \frac{x^2}{3} + \frac{x^4}{60} - \cdots \right]$$

$$= \sqrt{\frac{x}{2\pi}} \cdot \frac{2}{x} \left[x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots \right]$$

$$= \sqrt{\frac{2}{\pi x}} \sin x$$

4) c) If $x^3 + 2x^2 - x + 1 = a P_0(x) + b P_1(x) + c P_2(x) + d P_3(x)$ find the values of a, b, c, d.

Let $f(x) = x^3 + 2x^2 - x + 1$

$$f(x) = \frac{2}{5} P_3 + \frac{3}{5} P_1 + 2\left[\frac{1}{3} P_0 + \frac{2}{3} P_2\right] - P_1 + P_0$$

formula's for x^2, x^3

$$f(x) = \frac{2}{5} P_3(x) + \frac{4}{3} P_2(x) - \frac{2}{5} P_1(x) + \frac{5}{3} P_0(x)$$

$\therefore\ a = \frac{5}{3},\ b = -\frac{2}{5},\ c = \frac{4}{3},\ d = \frac{2}{5}$

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Module - III

5) a) Derive Cauchy - Riemann eqn in polar form.

Stmt :- if $f(z) = u(r,\theta) + iv(r,\theta)$ is analytic at a point z, then there exists 4 continuous first order partial derivatives $\frac{\partial u}{\partial r}, \frac{\partial u}{\partial \theta}, \frac{\partial v}{\partial r}, \frac{\partial v}{\partial \theta}$ and satisfies the eqns $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$ are called C-R eqns in polar form.

proof :- Let $f(z)$ be analytic at a point $z = re^{i\theta}$

hence by defn $f'(z) = \lim_{\delta z \to 0} \frac{f(z+\delta z) - f(z)}{\delta z}$

$$f'(z) = \lim_{\delta z \to 0} \frac{u(r+\delta r, \theta+\delta\theta) - u(r,\theta)}{\delta z} + i \lim_{\delta z \to 0} \frac{v(r+\delta r, \theta+\delta\theta) - v(r,\theta)}{\delta z} \quad \text{--- (1)}$$

$$\delta z = e^{i\theta}\delta r + ire^{i\theta}\delta\theta$$

case i) Let $\delta\theta = 0$ we get (1) ⇒

$$f'(z) = e^{-i\theta}\left[\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right] \quad \text{--- (2)}$$

case ii) Let $\delta r = 0$ we get (1) ⇒

$$f'(z) = e^{-i\theta}\left[\frac{1}{r}\frac{\partial v}{\partial \theta} - \frac{i}{r}\frac{\partial u}{\partial \theta}\right] \quad \text{--- (3)}$$

equating RHS of (2) & (3) and separating real and imaginary terms we get

$$\boxed{\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}} \quad \text{and} \quad \boxed{\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}}$$

These are Cauchy - Riemann eqns in the polar form.

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5) b) using Cauchy's residue theorem to evaluate the integral $\int_C \frac{z^2 dz}{(z-1)^2(z+2)}$ where C is the $\odot^{le}$ $|z|=3$.

here $f(z) = \frac{z^2}{(z-1)^2(z+2)}$

The poles $z=1$ is a ~~simple~~ pole of order $m=2$

$z=-2$ is a simple pole, both lies

within the $\odot^{le}$ $|z|=3$

$$R[m,a] = \frac{1}{(m-1)!} \lim_{z\to a} \frac{d^{m-1}}{dz^{m-1}} \{(z-a)^m f(z)\}$$

case i) Residue at $z=a=1$, $m=2$

$$R[2,1] = \lim_{z\to 1} \frac{d}{dz}\left\{\frac{z^2}{z+2}\right\}$$

$$= \lim_{z\to 1}\left[\frac{z^2+4z}{(z+2)^2}\right] = \frac{1+4}{9} = \frac{5}{9} = R_1$$

case ii) Residue at $z=a=-2$, $m=1$

$$R[1,-2] = \lim_{z\to -2}\left\{\frac{z^2}{(z-1)^2}\right\}$$

$$= \frac{(-2)^2}{(-2-1)^2}$$

$$= \frac{4}{3^2}$$

$$= \frac{4}{9} = R_2$$

hence cauchy's Residue theorem

$$\int_C f(z)\,dz = 2\pi i\,[R_1+R_2]$$

$$\int_C \frac{z^2 dz}{(z-1)^2(z+2)} = 2\pi i\left[\frac{5}{9}+\frac{4}{9}\right] = 2\pi i\,[1] = 2\pi i$$

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5) c) Find the BLT that transforms the points $z_1 = i$, $z_2 = 1$, $z_3 = -1$ on to the points $w_1 = 1$, $w_2 = 0$, $w_3 = \infty$ respectively.

>> here $z_1 = i$, $z_2 = 1$, $z_3 = -1$ and

$w_1 = 1$, $w_2 = 0$, $w_3 = \infty$ (or) $\frac{1}{w_3} = 0$

The required BLT is

$$\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$$

ie. $$\frac{(w-w_1)(w_2/w_3-1)}{(w/w_3-1)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$$

$$\frac{(w-1)(-1)}{(-1)(-1)} = \frac{(z-i)(1+i)}{(z+1)(1-i)}$$

$$(1-w) = \frac{2(z-i)}{(1-i)(z+1)}$$

$$w = 1 - \frac{2(z-i)}{(1-i)(z+1)} = \frac{(1-i)(z+1)-2(z-i)}{(1-i)(z+1)}$$

$$= \frac{(1+i)(1-z)}{(1-i)(1+z)}$$

$$= \frac{(1+i)^2(1-z)}{(1-i)(1+i)(1+z)}$$

$$= \frac{2i(1-z)}{2(1+z)}$$

$$w = \frac{i(1-z)}{}$$

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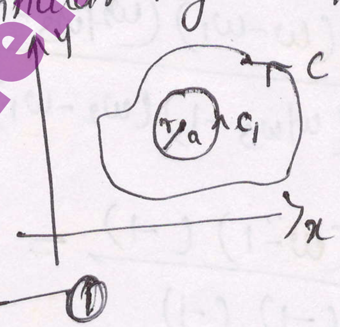
(OR)

6) a) State and prove Cauchy's integral formula.

Stmt :- If $f(z)$ is analytic inside and on a simple closed curve C and if 'a' is any point within C then $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-a)}\,dz$

Proof :- Since 'a' is a point within C, we shall enclose it by a circle C_1 with $z=a$ as centre and r as radius such that C_1 lies entirely within C. The fun $f(z)/z-a$ is analytic inside and on the boundary of the annular region b/w C and C_1



Now, as a consequence of Cauchy's theorem

$$\int_C \frac{f(z)}{(z-a)}\,dz = \int_{C_1} \frac{f(z)}{(z-a)}\,dz \quad \text{--- (1)}$$

The eqn of C_1 can be written in the form

$|z-a| = r$, $z-a = re^{i\theta}$, $z = a+re^{i\theta}$, $\theta \to 0$ to 2π

$dz = ire^{i\theta}d\theta$

RHS of (1)

$$\int_C \frac{f(z)}{z-a}\,dz = \int_{\theta=0}^{2\pi} \frac{f(a+re^{i\theta})}{re^{i\theta}}\, ire^{i\theta}d\theta$$

$$= i\int_0^{2\pi} f(a+re^{i\theta})\,d\theta$$

$r>0$, hence $r \to 0$ we get

$$= i\int_0^{2\pi} f(a)\,d\theta = i f(a)\cdot\theta\Big|_0^{2\pi} = 2\pi i f(a)$$

Thus $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-a)}\,dz$

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6) b) Given $u - v = (x-y)(x^2+4xy+y^2)$
find the analytic function $f(z) = u + iv$.

>> $u - v = x^3 + 3x^2y - 3xy^2 - y^3$ on simplification

$\therefore u_x - v_x = 3x^2 + 6xy - 3y^2$ —— ①

$u_y - v_y = 3x^2 - 6xy - 3y^2$

But $u_y = -v_x$ and $v_y = u_x$ by C-R eq^ns

$-v_x - u_x = 3x^2 - 6xy - 3y^2$ —— (2)

Let us solve for u_x and v_x from ① & ②

① + ② : $-2v_x = 6(x^2 - y^2)$

$v_x = 3(y^2 - x^2)$

① − ② : $2u_x = 12xy$

$u_x = 6xy$

we have $f'(z) = u_x + iv_x$

$f'(z) = 6xy + i\cdot 3(y^2 - x^2)$

putting $x = z,\ y = 0$ we get

$f'(z) = -3iz^2$

$\therefore f(z) = \int -3iz^2\, dz$

$= \left[-\cancel{3}i \cdot \frac{z^3}{\cancel{3}}\right] + C$

$= -iz^3 + C$

Thus $\boxed{f(z) = -iz^3 + C}$

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6) c) Discuss the transformation $w = z + (1/z)$

$$w = z + \frac{1}{z} \Rightarrow \frac{dw}{dz} = \left(1 - \frac{1}{z^2}\right) = 0 \quad \text{for } z = \pm 1$$

$w = u + iv$ and $z = r(\cos\theta + i\sin\theta)$

The given transformation can be written as

$$u = \left(r + \frac{1}{r}\right)\cos\theta \longrightarrow (1) \qquad v = \left(r - \frac{1}{r}\right)\sin\theta \longrightarrow (2)$$

Eliminating θ b/w (1) & (2)

$$\frac{u^2}{\left(r+\frac{1}{r}\right)^2} + \frac{v^2}{\left(r-\frac{1}{r}\right)^2} = 1 \longrightarrow (3)$$

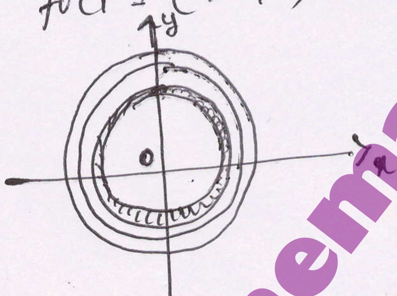
case i) Let $|z| = r = \lambda$ where λ is a non-zero const $(\neq 1)$

(3) $\Rightarrow$

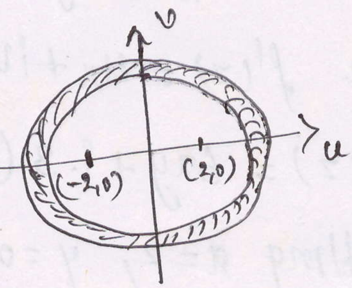
$$\frac{u^2}{\left(\lambda + \frac{1}{\lambda}\right)^2} + \frac{v^2}{\left(\lambda - \frac{1}{\lambda}\right)^2} = 1 \longrightarrow (4) \quad \text{ellipse in } w\text{-plane}$$

eccentricity: $e^2 = \dfrac{a^2 - b^2}{a^2}$ $\quad \therefore ae = 2$

foci $= (\pm 2, 0)$ and $w = \pm 2$.



z-plane



w-plane

case ii) when $r = \lambda = \text{constant} > 1$

as $\lambda \to 1$ to ∞, $1/\lambda \to 1$ to 0,

by the two confocal ellipses

$$\frac{u^2}{\left(\lambda_2 + \frac{1}{\lambda_2}\right)^2} + \frac{v^2}{\left(\lambda_2 - \frac{1}{\lambda_2}\right)^2} = 1$$

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case iii) Eliminating r b/w the eqns ① and ② we get

$$\frac{u^2}{4\cos^2\theta} - \frac{v^2}{4\sin^2\theta} = 1 \quad \text{when } \theta = \alpha \ (0 < \alpha < \pi/2) \text{ the above eq}^n$$

represents a hyperbola in the w-plane.

$$e^2 = \frac{\cos^2\alpha + \sin^2\alpha}{\cos^2\alpha} = \sec^2\alpha \Rightarrow e = \sec\alpha, \ \text{foci} = (\pm 2, 0)$$

hence the image of the radial lines $\theta = \alpha$ for varying α in the z-plane is a family of confocal hyperbolas with vertices at the points $w = \pm 2\cos\alpha$ and foci at the points $\omega = \pm 2$.

Module - IV

7) a) Derive mean and standard deviation of the binomial distribution.

$$\text{Mean}(\mu) = \sum_{x=0}^{n} x\, p(x)$$

$$\mu = \sum_{0}^{n} x \cdot {}^nC_x\, p^x q^{n-x}$$

$$= \sum_{0}^{n} x \cdot \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{0}^{n} \frac{n(n-1)!}{(x-1)!\,(n-x)!} p \cdot p^{x-1} q^{n-x}$$

$$= np \sum_{1}^{n} \frac{(n-1)!}{(x-1)!\,[(n-1)-(x-1)]!} p^{x-1} q^{(n-1)-(x-1)}$$

$$= np \sum_{1}^{n} {}^{(n-1)}C_{(x-1)}\, p^{x-1} q^{(n-1)-(x-1)}$$

$$= np\,(p+q)^{n-1}$$

$$= np\,(1)$$

$$= np$$

$$\therefore \text{Mean}(\mu) = np$$

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$$\text{Variance } (V) = \sum_{x=0}^{n} x^2 p(x) - M^2$$

$$\text{Now } \sum_{x=0}^{n} x^2 p(x) = \sum_{x=0}^{n} [(x(x-1)+x)\, p(x)$$

$$= \sum_{0}^{n} x(x-1)\, p(x) + \sum_{0}^{n} x\, p(x)$$

$$= \sum_{0}^{n} x(x-1)\, {}^{n}C_{x}\, p^{x} q^{n-x} + np$$

$$= \sum_{0}^{n} \frac{n(n-1)(n-2)!}{(x-2)!\,(n-x)!}\, p^2 p^{x-2} q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n} {}^{(n-2)}C_{x-2}\, p^{x-2} q^{(n-2)-(x-2)} + np$$

$$= n(n-1)p^2 (q+p)^{n-2} + np$$

$$= n(n-1)p^2 + np$$

$$\therefore V = npq$$

$$S.D(\sigma) = \sqrt{V} = \sqrt{npq}$$

Hence $\boxed{M = np}$ $\boxed{S.D(\sigma) = \sqrt{npq}}$

7) b) In a certain factory turning out razor blades, there is a small chance of 0.002 for a blade to be defective. The blades are supplied in packets of 10. Use poisson distribution to calculate the approximate number of packets containing.

i) no defective

ii) one defective

iii) two defective blades, in a consignment of 10,000 packets.

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$P = \underline{prob}$ of a defective blade $= 0.002$

In a packet of 10,

$$m = np = 10 \times 0.002 = 0.02$$

$$p(x) = \frac{m^x e^{-m}}{x!} = \frac{e^{-0.02}(0.02)^x}{x!}$$

Let $f(x) = 10{,}000\, p(x)$ also $e^{-0.02} = 0.9802$

$$\therefore \quad f(x) = \frac{9802\,(0.02)^x}{x!}$$

i) $\underline{prob}$ of no defective $f(0) = 9802$

ii) $\underline{prob}$ of one defective $f(1) = 196$

iii) $\underline{prob}$ of two defective $f(2) = 2$

7)

c) The joint $\underline{prob}$ distribution for two random variables X and Y is given below:

X \ Y	−2	−1	4	5
1	0.1	0.2	0.0	0.3
2	0.2	0.1	0.1	0.0

Determine

i) marginal distribution of X and Y

ii) Covariance of X and Y

iii) Correlation b/w X and Y.

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i) marginal distribution of x and y

x	1	2
f(x)	0.6	0.4

y	−2	−1	4	5
g(y)	0.3	0.3	0.1	0.3

$E(x) = \Sigma x f(x) = 1.4$ $\qquad E(y) = \Sigma y\, g(y) = 1$

$E(xy) = \Sigma xy J_{ij} = 0.9$

ii) $Cov(x, y) = E(xy) - E(x)E(y) = -0.5$

iii) $\sigma_x^2 = E(X^2) - [E(x)]^2$

$E(X^2) = 2.2 = \Sigma x^2 f(x)$

$\sigma_x^2 = 2.2 - (1.4)^2 = 0.24$

$\boxed{\sigma_x = 0.49}$

$\sigma_y^2 = E(y^2) - [E(y)]^2$

$E(y^2) = \Sigma y^2 g(y) = 10.6$

$\sigma_y^2 = 10.6 - (1)^2 = 9.6$

$\boxed{\sigma_y = 3.1}$

$\rho(x, y) = \dfrac{cov(x,y)}{\sigma_x \sigma_y} = -0.3$

$\boxed{\rho(x,y) = -0.3}$

(OR)

8) a) The average daily turn out in a medical store is RS. 10,000 and the net profit is 8%. if the turn out has an exponential distribution, find the probability that the net profit will exceed Rs 3000 each on two consecutive days.

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Let x be the random variate of the medical store

Since the variate is exponential the p.d.f

$$f(x) = \alpha e^{-\alpha x}, \quad x > 0$$

$$mean = \frac{1}{\alpha}$$

$$10,000 = \frac{1}{\alpha} \Rightarrow \alpha = \frac{1}{10,000} = 0.0001$$

hence $\underline{f(x) = 0.0001\, e^{-0.0001x}}, \quad x > 0$

Let A be the amount for which profit 8%.

$$\Rightarrow A \cdot \frac{8}{100} = 3000$$

$$A \cdot 0.08 = 3000$$

$$A = \frac{3000}{0.08} = 37500$$

$$\boxed{A = 37500}$$

<u>Prob.</u> of profit exceeding RS. 3000 is equal to

$$= 1 - \underline{prob}\,(profit \leq 3000)$$

$$= 1 - \underline{Prob}\,(store \leq 37500)$$

$$= 1 - \int_0^{37500} f(x)\,dx$$

$$= 1 - \int_0^{37500} 0.0001\, e^{-0.0001x}\,dx$$

$$= 1 - \cancel{0.0001} \left. \frac{e^{-0.0001x}}{-\cancel{0.0001}} \right|_0^{37500}$$

$$= 1 + e^{-0.0001x}\Big|_0^{37500}$$

$e^{-3.75} = 0.0235$ Single day

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8) b) The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the no. of students whose marks will be

i) less than 65

ii) more than 75

iii) b/w 65 and 75. given $\phi(1) = 0.3413$

>> Let x represents the marks of students.

By data $\mu = 70$, $\sigma = 5$,

S.N.V $z = \frac{x-\mu}{\sigma} = \frac{x-70}{5}$

i) If $x = 65$, $z = -1$,

$\therefore P(z<-1) = P(z>1)$
$= 0.5 - \phi(1)$
$= 0.5 - 0.3413$
$= 0.1587 \times 1000 = 159$

no. students scoring less than 65 marks 159 students

ii) If $x = 75$, $z = 1$

$P(z>1) = 0.5 - \phi(1) = 0.5 - 0.3413 = 0.1587$
$= 0.1587 \times 1000 = 159$

$\therefore$ no. students scoring more than 75 marks 159 students

iii) $P(-1<z<1) = 2P(0<z<1) = 2\phi(1) = 0.6826$
$= 0.6826 \times 1000 = 683$

$\therefore$ no. of students scoring marks b/w 65 and 75 is 683 students.

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8) c) The joint distribution of two random variables X and Y is as follows.

$X \backslash Y$	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

compute

i) $E(X)$, $E(Y)$

ii) $E(XY)$ iii) $cov(X,Y)$

iv) $\rho(X,Y)$

x	1	5
$f(x)$	1/2	1/2

y	-4	2	7
$g(y)$	3/8	3/8	1/4

$E(X) = \Sigma x f(x) = 3$ $E(Y) = \Sigma y g(y) = 1$ $E(XY) = \Sigma xy p_{ij}$
$= 1.5$

$cov(X,Y) = E(XY) - E(X)E(Y) = -1.5$

$\sigma_x^2 = E(X^2) - (E(X))^2$

$\sigma_x = 2$ $\sigma_y = 4.330$

$$\rho(X,Y) = \frac{cov(X,Y)}{\sigma_x \sigma_y} = \underline{\underline{-0.1732}}$$

Module - V

9) a) Explain the terms:

i) Null hypothesis:-

The hypothesis formulated for the purpose of its rejection under the assumption that it is true is called null hypothesis.

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ii) confidence intervals:-

The interval in which the population parameter is supposed to lie is called the confidence interval for that population parameter. The end point of this interval are called the confidence limits.

iii) Type I and Type II errors:-

In a test process there can be two situations leading to two types of errors as given below

Type I error: If a hypothesis is rejected while it should have been accepted is known as Type I error.

Type II error: If a hypothesis is accepted while it should have been rejected is known as Type II error.

9)
b) A certain stimulus administered to each of the 12 patients resulted in the following change in the blood pressure 5, 3, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 can it be concluded that the stimulus will increase the blood pressure. ($t_{.05}$ for 11 df 2.201)

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Let H_0 – stimulus administered to all the 12 patients will increase the blood pressure.

$$t = \frac{\bar{x} - \mu}{s} \sqrt{n} \quad \text{——— (1)}$$

$$\bar{x} = \frac{1}{n} \Sigma x = \frac{1}{12} [5+2+8-1+0+3+6-2+1+5+0+4]$$

$$\bar{x} = \frac{31}{12} = 2.5833$$

$$s^2 = \frac{1}{n-1} \left[\Sigma x^2 - \frac{1}{n} (\Sigma x)^2 \right]$$

$$= \frac{1}{11} \left\{ 185 - \frac{1}{12} (31)^2 \right\}$$

$$s^2 = 9.538$$

$$\therefore \boxed{s = 3.088}$$

let us suppose that, we can take $\mu = 0$

$$(1) \Rightarrow t = 2.9 > 2.201$$

H_0 is rejected.

9)

c) Show that the markov chain whose transition probability matrix $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$

is irreducible.

Also find the corresponding stationary probability vector.

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we shall show that P is a regular stochastic matrix, For convenience we shall write the given matrix in the form,

$$P = \frac{1}{6}\begin{bmatrix} 0 & 4 & 2 \\ 3 & 0 & 3 \\ 3 & 3 & 0 \end{bmatrix}$$

$$P^2 = \frac{1}{36}\begin{bmatrix} 18 & 6 & 12 \\ 9 & 21 & 6 \\ 9 & 12 & 15 \end{bmatrix}$$

Since all the entries in P^2 are positive we conclude that the t.p.m P is regular.

Hence the markov chain having t.p.m P is irreducible.

Next we shall find the fixed prob vector of P.

$v = (x, y, z)$ we shall find v such that

$vP = v$ where $x+y+z = 1$

$$\Rightarrow [x, y, z] \cdot \frac{1}{6}\begin{bmatrix} 0 & 4 & 2 \\ 3 & 0 & 3 \\ 3 & 3 & 0 \end{bmatrix} = [x, y, z]$$

$$\Rightarrow \frac{1}{6}[3y+3z,\ 4x+3z,\ 2x+3y] = [x, y, z]$$

$3y+3z = 6x$, $4x+3z = 6y$, $2x+3y = 6z$

using $x+y+z=1$, we get

$x = 1/3$, $y = 10/27$, $z = 8/27$

Thus $v = (1/3, 10/27, 8/27)$ is the required stationary probability vector.

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10) (OR)

a) In an elementary School examination the mean grade of 32 boys was 72 with a standard deviation of 8 while the mean grade of 36 girls was 75 with a standard deviation of 6. Test the hypothesis that the performance of girls is better than boys.

By data Boys : $\bar{x}_1 = 72$, $\sigma_1 = 8$, $n_1 = 32$

girls ; $\bar{x}_2 = 75$, $\sigma_2 = 6$, $n_2 = 36$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$$

$Z = -1.73$

$|Z| = 1.73$

$\therefore |Z| = 1.73 \begin{cases} > Z_{0.05} = 1.645, \text{ Ho rejected (one tailed test)} \\ < Z_{0.01} = 2.33, \text{ Ho accepted.} \end{cases}$

10)

b) Four Coins are tossed 100 times and the foll^ng results were obtained.

No of heads	0	1	2	3	4
frequency	5	29	36	25	5

Fit a binomial distribution for the data and test the goodness of fit. ($\chi^2_{0.05} = 9.49$ for 4 df)

Let Ho – goodness of fit

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$$\text{Mean}(m) = \frac{\Sigma fx}{\Sigma f} = \frac{196}{100} = 1.96$$

$m = np \quad n = 4$

$1.96 = 4p \Rightarrow p = \frac{1.96}{4} = 0.49 \quad q = 1 - p = 0.51$

$p(x) = n_{c_x} p^x q^{n-x} = 4_{c_x} (0.49)^x (0.51)^{4-x}$

$f(x) = 100 p(x)$

$f(0) = 7, \; f(1) = 26, \; f(2) = 37, \; f(3) = 24, \; f(4) = 6$

O_i	5	29	36	25	5
E_i	7	26	37	24	6

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{4}{7} + \frac{9}{26} + \frac{1}{37} + \frac{1}{24} + \frac{1}{6}$$

$$= 1.15$$

Thus $\chi^2 = 1.15 < 9.49$

H_0 is accepted.

10) c) Every year, a man trades for his car for a new car, if he has maruthi, he trade it for a Ford. If he had a Ford. he trade it for a Hyundai. However, if he had a Hyundai, he is just as likely to trade it for a new hyundai as to trade it for a maruthi (or) a Ford. In 2014 he bought his first car which was a Hyundai. Find the prob that he has a) 2016 Ford b) 2016 Hyundai c) 2016 maruthi

>> The State Space of the system $\{A, B, C\}$

$A = \text{maruthi} \quad B = \text{Ford} \quad C = \text{Hyundai}$

The associated t.p.m is

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

In 2014 as the first year, 2016 is to be ragarded as 2 years.

we need to compute P^2

$$P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{bmatrix}$$

$$\therefore P^2 = \begin{array}{c} \\ A \\ B \\ C \\ \\ \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \left[\begin{array}{ccc} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{array}\right] \\ \begin{array}{ccc} M & F & H \end{array} \end{array} = \begin{bmatrix} a_{11}^2 & a_{12}^2 & a_{13}^2 \\ a_{21}^2 & a_{22}^2 & a_{23}^2 \\ a_{31}^2 & a_{32}^2 & a_{33}^2 \end{bmatrix}$$

a) 2016 Ford $\rightarrow a_{32}^{(2)} = \frac{4}{9}$

b) 2016 Hyundai $\rightarrow a_{33}^{(2)} = \frac{4}{9}$

c) 2016 maruthi $\rightarrow a_{31}^{(2)} = \frac{1}{9}$

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