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OPERATIONS RESEARCH

Designed for Computer Science Students

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Introduction, Linear Programming

Learning objectives

After Studying this chapter, you should be able to

- understand the need of studying 'OR'
- know the historical perspective and applications of OR
- formulate a problem as a Linear Programming (LP) Problem
- solve a Linear Programming Problem using graphical method
- understand the special cases in graphical method

1.1 INTRODUCTION

Operations Research (OR) is an art and science concerned with the efficient allocation /utilization of limited resources. The art lies in the ability to depict the available resources in a well–defined mathematical model for a given situation. The science consists in the derivation of computational methods for solving such models. Whether it is a company, a farm, or even a domestic kitchen, resources of men, machine, material, money etc.. have to be utilized in a most efficient manner.

In this regard, the management has to constantly analyze the existing situation and make proper decisions.

Decision–making is always a complex activity as it affects the decision–maker as well as others. For example a student has to decide the course he / she should take for study and the decision will affect not only the student but also everybody related to his / her life and activities. The same principle applies to a person seeking employment where he / she has to chose the job or service. Therefore, one has to develop his / her talents in such a way that he / she is in a position to take a correct decision at a proper time. Effective decision making depends on many factors such as economic, social and political. For example starting a new firm/business at a place would depend on economic factors such as construction costs, labour availability/costs, availability of raw materials, transportation costs, taxes, energy, pollution control costs, etc.

Decision making in business and industry is extremely challenging as it affects many people from a wide spectrum of society. Generally decisions are taken with the help of past experience and instructions but some formal system is needed to determine an effective course of action. Operations Research (also known as Optimization Techniques, Management Science and by few other names) provides a quantitative technique or scientific approach to the executives for making better decisions for operations under their control. In other words, OR provides a scientific approach to problem solving.

OR basically helps in determining the best (optimum) solution to problems where decisions need to be taken under the restriction of limited sources. It is possible to convert any real life problem into a mathematical model. The basic feature of OR is to formulate a real world problem as a mathematical model. In general, to management of organisations concerned with lowering labour costs or production costs or transportation costs to achieve higher profits, OR can be very usefully employed to minimize the costs/ maximize the profits (Optimization).

1.2 Why to Study Operations Research? (Importance of Studying OR)

OR basically helps in determining the best (optimum) solution to problems where decision has to be taken under the restriction of limited resources. Any organization involving operations (transportation, job allocation, marketing) want to lower their operation costs to achieve higher profits, OR can be very usefully employed to this kind of real life problems.

With computers moving up the corporate ladder, the managers are increasingly using the operations research techniques for the purpose of decision – making with a view to arrive at optimal decisions. Thus, an understanding of various important techniques which can be used to aid the managerial decision making process is desirable for engineers / managers.

Industry has become quite aware of the potential of OR as a technique and many industrial and business houses have OR teams working to find solutions to their problems.

1.3 Definitions of Operations Research

Operations Research has been defined so far in various ways and it is perhaps still too young to be defined in some authoritative way. Students must understand that it is not possible to give uniformly acceptable definition of OR. A few opinions about the definitions of OR are,

"OR is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control" **– Morse and Kimbol**

"OR is the scientific method of providing executives an analytical and objective basis for decision" **– P. M. S. Blackett**

"Operations Research is a scientific approach to problem solving for Executive Management" **– H. M. Wanger**

"Operations Research is concerned with scientifically deciding how best to design and operate machine systems usually under conditions requiring the allocation of scarce resources" **– OR Society of America**

"OR is the scientific knowledge through inter disciplinary team efforts for the purpose of determining the best utilization of limited resources" **– H. A. Taha**

"Operations research is an art of winning a war without actually fighting" **– Aurther Clark**

"Operations research is the application of the methods of science to complex problems arising in the direction and management of large systems of men, materials and money in industry, business, government and defense" **– Operations society of Great Britan**

The above discussed definitions are given by various people at different times and stages of development of operations research lays emphasis on

i. ***OR being a scientific technique***

ii. ***It is a problem solving technique***

iii. ***It is for the use of executives who have to take decisions for the organizations.***

A close observation reveals that, almost all the definitions are conveying the same meaning.

1.4 Origin of Operations Research (Historical Development of OR)

The term, operations research was first coined in 1940. This new science came into existence in a military contest. During World War–II, Military Management in England called upon a team of scientists to study the strategic and tactical problems related to air and land defense of the country. They were having limited resources and it was necessary to decide upon the most effective utilization of them. (Effective ocean transportation, effective bombing etc). **Mc Closky** and **Trebthen** of Bowdsey, **United Kingdom** used the term Operations Research in 1940 to describe this new science.

The OR teams were not actually engaged in military operations and fighting the war. But, they were only instrumental and advisors in wining the war by providing a good intellectual support to the strategic initiatives of the military commands (that is, "An art of wining the war without actually fighting"). As the team was dealing with research on military operations, the work of this team of scientists was named as Operations Research in England.

Following the end of war, the success of military teams attracted the attention of industrial managers who were seeking solutions to their complex executive type problems. Thus, it started spreading throughout the world and Society of Operations Research was formed in United States.

Today the impact of OR is felt on many areas. A large number of management consulting firms are currently engaged in OR activities.

1.5 The Nature of Operations Research

As it's name implies, operations reasesrch involves" Research on Opeartions ". Thus, OR is applied to problems that concern how to conduct and co-ordinate the operations (activities / tasks) within an organization.

The nature of organization is essentially immaterial and in fact OR has been applied extensively in diversified areas such as manufacturing, transporetation, construction, telecommunication, financial planning, health care, the military and public services to name just a few.

The research part of the name means that operations research uses an approach that resembles the way research is carried out in the established scientific fields. To a considerable extent, the scientific method is used to investigate the problem of concern.

In particular,the process being by carefully observing and formulating the problem, including gathering all the relavant data. The next step is to construct a scientific model that attempts to abstract the essence of the real problem. It is then hypothesised that this model is a scientifically precise representation of the essential features of the situation that the solutions obtained from the model are also valid for the real problem. Next to this, suitable experiments are conducted to test this hypothesis, modify if as needed and eventually verify some form of the hypothesis known as validation. Thus, in a certain sense OR involves creative scientific research into the fundamental properties of operations.

1.6 Impact of Operations Research (Applications / Scope of OR)

Operations Research has had a notable impact on improving the efficiency of numerous organisations around the world. In the process, OR has made a significant contribution in enhancing the economy of the various countries.There are a few dozen member countries in the International Federation of Operational Research Societies (IFORS), with each country having a national OR society. Both Europe and Asia have federations of OR societies to co-ordinate the holding of international conferences and publishing international journals in these continents. In addition, the Institute for Operations Research and the Management Sciences (INFORMS) is an international OR society.

The scope of Operation Research is far and wide. In recent years OR has successfully entered many different areas of research in defense, government, service organizations and Industry. Not all applications of OR can be listed, as OR is a tool finding new applications every day. OR has had an impressive impact on improving efficiency of various organisations across the world.

1.6.1 Areas of Applications of Operations Research

An outline of applications of Operations Research.

i. *Optimal allocation of resources.*

ii. *Finding the optimal trajectories of missile vehicles.*

iii. *Design of aircraft and aerospace structure with high strength and low–weight.*

vi. *Design of structures like frames, bridges, chimneys, dams etc., at minimum cost.*

v. *Selection of best location for an organization.*

vi. *To find shortest route taken by a salesman, visiting different cities.*

vii. *Optimum design of electrical / cable networks.*

viii. *Optimum production planning, controlling and scheduling.*

ix. *Selection of an optimum strategy.*

x. *In finding solutions for various business problems.*

These can be summarized / classified into various areas or fields as follows.

1. **Production Management**
   a. Scheduling and requesting the production run by proper allocation of machines
   b. Obtaining optimum product mix.
   c. Selection, location and decision of the sites for the production plans.
2. **Purchasing Decision**
   a. Inventory (Stock) management
   b. Optimal re–ordering
3. **Facilities Planning** ***Management***
   a. Transportation, loading and unloading
   b. Planning warehouse locations
   c. Factory/building location and size decisions
4. **Construction Management**
   a. Location of resources to different projects
   b. Work force/labour planning
   c. Project management
5. **Personnel Management**
   a. Forecasting the man–power requirements, recruitment policies and job assignments
   b. Selection of suitable personnel with due consideration for age and skills etc.

6. Research and Development

a. Reliability and evaluation of alternative projects

b. Control of developed projects

c. Determination of time and cost requirements

Apart from the above applications, its use has newly extended to a wide range of problems, such as the problems of communication, information, agriculture and national planning.

Indian Railways, Indian Airlines, Hindustan Lever Limited, Tata Iron and Steel Company, Fertilizer Corporation of India, Life Insurance Corporation of India are few industries / organizations which are implemented the OR methodology.

1.7 Defining the Problem and Gathering Data (Phases/Methodology of OR)

The systematic methodology developed for an OR study deals with the problems involving conflicting multiple objectives, policies and alternatives.

The OR approach to problem solving consists of the following steps

i. **Formulation of the Problem:** *It involves description of the objective, identification of the decision variables and constraints of the system.*

ii. **Construction of Mathematical Model:** *After formulating the problem, the next step is to construct a model for the system under study. It is usually a mathematical model. A mathematical model consists of a set of equations that describe the system.*

iii. **Deriving the Solution from the Model:** *Once the Mathematical model is formulated, the next step is to determine the values of decision variables that optimize the given objective function. This deals with mathematical calculations for obtaining the solution to the model.*

iv. **Validation of the Model:** *A model is valid if it can give reasonable predication of the performance of the system. The validity of a model is tested by comparing its performance with previous data available to the system. Comparison should reveal favorable results.*

v. **Controlling the Solutions:** *After testing the model and its solution, the next step is to establish control over the solution, by proper feedback of the information on variables which deviate significantly. In case of any deviation the model may be modified accordingly.*

vi. **Implementation of the Final Results:** *Finally, the tested results of the model are implemented to work. For this, solution obtained above should be translated into operating procedures that can be easily understood and applied by those who control the operations.*

1.7.1 Mathematical Model of OR

It involves description of the objective, identification of the decision variables and constraints of the problem. These variables are represented by a set of mathematical equations.

Example

Let us consider a company producing desktops, laptops with the objective of achieving maximum profit, then the decision variables are the number of laptops, desktops to produce. The limitation on market requirements, resources available are the constraints.

1.7.2 Testing the Model

After computing and deriving the solution from the model, it is once again tested as a whole for the errors if any. A model may be said to be valid if it can provide a reliable production of the system's performance, a good practitioner of operations research believes that this model be applicable for a longer time and thus updates model time to time by considering the past, present and future specifications of the problems.

1.7.3 Preparing to Apply the Model

This establishes control over the solution with desired degrees of satisfaction. The model requires immediate modification as soon as the control variables change significantly, otherwise the model goes out of control. This action is often referred as sensitivity analysis.

As the conditions will be constantly changing, the model and solution may not remain valid for a long time.

1.7.4 Deriving the Solution from the Model

From the formulated Mathematical model, it is required to determine the values of decision variables that optimizes (minimizes or maximizes) the given objective function as specified. This deals with the mathematical computations in obtaining the solution to the model.

1.8 Characteristics of OR (Features of OR)

A model does not always have the characteristic of being a yardstick – it can be explanatory rather than merely descriptive. Following are the main characteristics that a good OR model should have,

i. ***The number of assumptions made should be as few as possible.***

ii. ***The model should be simple and coherent. The number of variables utilized by it should be small in number.***

iii. ***An OR model should take into account new formulations without having to make any significant changes.***

iv. ***It should be adaptable to parametric type of treatments.***

v. ***It should be easy and economical to construct.***

vi. ***Operations Research is for Operations economy.***

1.9 Limitations of OR Models

i. *There are certain problems that a decision–maker may have to solve only once. Constructing a complex OR model for solving such problems is often too expensive when compared to the cost of other less sophisticated approaches available.*

ii. *Some times the model may not represent the "Real World problem" in where decisions have to be made.*

iii. *If the basic data is subjected to frequent changes, modification of OR is costlier*

iv. *Some models are so complex that they require computers to solve and some solutions are difficult to explain to the managers.*

v. *It cannot account any factor that cannot be quantified such as human behavior.*

1.10 Linear Programming

Decision making has always been very important in the business and industrial world. Every organization usually faces the problem of allocation of resources. The resources include men, machine, material, information etc. Most of the decisions made are subject to constraints. Linear programming is one of the most versatile, popular and widely used quantitative techniques. Optimum scheduling of inter dependent activities can be determined, in view of available resources.

The word "linear" stands for indicating that all relationships involved in a particular problem are linear. Programming is another word of planning. The criterion of optimality generally is either performance, profit, time, cost, distance etc..

Definition

Linear programming is an optimization technique for finding an optimal (maximum or minimum) value of a function, called objective function, of several independent variables. The variables being subject to constraints (or restrictions) expressed as equations or inequalities.

Components (elements) of an LP model

The general structure of LP models consists of three basic elements or components.

i. **Decision Variables:** *These are the unknowns to be determined subject to the given constraints, usually these are denoted by x_1, x_2,--- x_n.*

ii. **Objective Function:** *A function known as objective function is expressed in terms of the decision variables and it is usually denoted by Z.*

iii. **Constraints:** *There are always certain limitations (constraints) on the use of resources, e.g. labour, raw material, money etc., These constraints limit the value of objective function.*

1.11 Steps/Guidelines in Formulation of a Linear Programming Problem

The procedure for mathematical formulation of a LP problem consists of the following steps.

Step 1: Write down the decision variables of the problem. (say x_1 , x_2 ......x_n)

Step 2: Formulate the objective function to be optimized as a linear function of the decision variables.

Step 3:

Formulate the other conditions of the problem such as resource limitation, market constraints etc as linear inequalities or equations in terms of decision variables.

Step 4:

Add the non–negativity constraint so that the negative values of decision variables do not have any valid physical interpretation.

Note: *The objective function, the group of constraints including the non–negative constraint forms a linear programming problem.*

1.12 Mathematical Formulation of an LPP

The general formulation of a linear programming problem can be stated as follows,

In order to find n decision variables x_1, x_2....x_n to maximize or minimize the objective function

$Z = C_1x_1 + C_2 x_2 + \ldots + C_n x_n$ .... **(A)**

Satisfying *m* constraints

$$a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n \ (\leq \text{or} = \text{or} \geq)\ b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n \ (\leq \text{or} = \text{or} \geq)\ b_2$$

$$\vdots \qquad\qquad \textbf{(B)}$$

$$\vdots\ldots\ldots\ldots\ldots\ldots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n \ (\leq \text{or} = \text{or} \geq)\ b_m$$

Where, constraints may be in the form of inequality $\geq$, $\leq$ or even in the form of an equation (=) and finally satisfying the non–negative constraint $x_1 \geq 0_1$, $x_2 \geq 0$......$x_n \geq 0$.

Z is the objective function.

C_1, C_2.....C_n are coefficients in the objective function.

a_{11}, a_{22}..... a_{1n} a_{21},a_{22}..... a_{2n} etc., are the coefficients of constraints.

b_1,b_2..... b_m are resources.

1.13 Assumptions in Linear Programming Problem

The important assumptions in linear programming problem are,

Assumptions made in linear programming models

Linearity: It is assumed that decision variables are of the first power. There is no provision for higher powers like squares and cubes in any of the equations and inequalities.

Divisibility: Values of the decision variables are allowed to be fractions and need not be integers alone.

Deterministic Parameters: It is assumed that the values of the parameters are known and are constant. This means that the model assumes a static state.

It is pertinent to point out that in real life situation, there are probabilistic model parameters.

Non negativity: All decision variables must take on non-negative values.

Proportionality: It is assumed that the output obtained from a resource is proportional to the input of the resource.

Additivity: The linearity concept also implies that the total measures of the objective function and the total usage are additive in nature.

Independence of variables: It is assumed that the various components of the model work independently.

Existence of an optimal solution: There exists an optimal solution to the objective function, taking into consideration the identified constraints

1.14 Applications of LPP

Linear programming is a widely used field of optimization for several reasons. Many practical problems in operations research can be expressed as linear programming problems. Certain special cases of linear programming, such as network flow problems and multicommodity flow problems are considered important enough to have generated much research on specialized algorithms for their solution. A number of algorithms for other types of optimization problems work by solving LP problems as sub-problems.

Linear programming was heavily used in the early formation of microeconomics and is currently utilized in company management, such as planning, production, transportation, technology and other issues. Although the modern management issues are ever-changing, most companies would like to maximize profits or minimize costs with limited resources. Therefore, many issues can be characterized as linear programming problems.

In practice, LPP has proved to be one of the most widely used techniques of managerial decision making in business, industry and numerous other fields. To mention just a few,

i. Agriculture sector
ii. Miltary operations
iii. Production Management
iv. Financial Management
v. Investment Planning
vi. Marketing Management
vii. Allocation of limited resources
viii. Administartive management

Worked Examples

1. *A company manufactures FM radios and calculators. The radios contribute ₹100 per unit and calculators ₹150 per unit as profit. Each radio requires 4 diodes and 4 resistors while each calculator requires 10 diodes and 2 resistors. A radio takes 12 minutes and calculator takes 9.6 minutes of time on the company electronic testing machine and the product manager estimates that 160 hours of test time is available. The firm has 8000 diodes and 3000 resistors in the stock. Formulate the problem as LPP.*

Solution:

Let x_1, x_2 be the number of radios and calculators to be manufactured.

Profit on one unit of radio is ₹100/–. So profit on x_1 number of units = $100x_1$

Profit on one unit of calculator is ₹150/–. So profit on x_2 number of units = $150x_2$

Total profit = $100x_1 + 150x_2$

Let the total profit be represented by Z_{max}, then $Z_{max} = 100x_1 + 150x_2$, is the objective function

Subject to the constraints,

Number of diodes required for one radio is 4, so total number of diodes for x_1 number of radios will be $4x_1$.

Number of diodes required for one calculator is 10, so total number of diodes for x_2 number of calculators will be $10x_2$.

Hence, the diodes constraint is $4x_1 + 10x_2 \leq 8000$ (as the total number of diodes cannot exceed 8000 units, available in the stock)

Similarly the resistors constraint is, $4x_1 + 2x_2 \leq 3000$ and the time constraint is,

$12x_1 + 9.6x_2 \leq 9,600$ (160 hours = 160 × 60 minutes,)

Thus, the formulation for the given problem is,

$Z_{max} = 100x_1 + 150x_2$

Subject to

$4x_1 + 10x_2 \leq 8000$

$4x_1 + 2x_2 \leq 3000$

$12x_1 + 9.6x_2 \leq 9600$

$x_1, x_2 \geq 0$ (non negative constraint)

Non negative constraint is a default constraint which indicates that x_1, x_2 cannot be negative in other words producing negative number of radios and calculators is meaning less.

2. *A computer company manufactures lap tops and desktops that fetches profit of ₹700/– and 500/–unit respectively. Each unit of laptop takes 4 hours of assembly time and 2 hours of testing time while each unit of desktop requires 3 hours of assembly time and 1 hour for testing. In a given month the total number of hours available for assembly is 210 hours and for inspection is 90 hours. Formulate the problem as LPP in such a way that the total profit is maximum.*

Solution:

i) Objective Function

Let x_1 be the number of laptops, x_2 be the number of desktops.

Profit on one unit of laptop is ₹700/–. So the profit on x_1 number of units = $700x_1$

Profit on one unit of desktop is ₹500/–. So the profit on x_2 number of units = $500x_2$

Total profit = $700x_1 + 500x_2$

Let the total profit be represented by Z_{max}, then the objective function is $Z_{max} = 700x_1 + 500x_2$ subject to,

ii) Constraints

a) Assembly time

Number of assembly hours per one laptop is 4 hence, assembly time for x_1 number of laptops is $4x_1$ and number of assembly hours per one desktop is 3 so, the assembly time for x_1 number of desktops is $3x_2$. The total assembly time is $4x_1 + 3x_2$ which should be less than 210 hours. This constraints is represented as,

$4x_1 + 3x_2 \leq 210$

b) Inspection time

Number of inspection hours per one laptop is 2 hence, inspection time for x_1 number of laptops is $2x_1$ and number of inspection hours per one desktop is 1 so, the inspection time for x_1 number of desktops is $1x_2$. The total inspection time is $2x_1 + 1x_2$ which should be less than 90 hours. This constraints is represented as,

$2x_1 + 1x_2 \leq 90$

Thus, the formulation for the given problem is,

$Z_{max} = 700\ x_1 + 500\ x_2$ subject to the following constrains

$4x_1 + 3x_2 \leq 210$ (Assembly time)

$2x_1 + x_2 \leq 90$ (Inspection time)

$x_1, x_2 \geq 0$ (Non–negativity constraint)

Prototype Example

3. *The Whitt window company is a company with only three employees which makes two different kinds of hand crafted windows: a wood–framed and an aluminium–framed window. They earn \$60 profit for each wood–framed window and \$30 profit for each aluminum–framed window. Doung makes the wood frames, and can make 6 per day. Linda makes the aluminum frames, and can make 4 per day. Bob forms and cuts the glass, and can make 48 square feet of glass per day. Each wood–framed window uses 6 square feet of glass and each aluminum–framed window uses 8 square feet of glass.*

The company wishes to determine how many windows of each type to produce per day to maximize the profit. Formulate a linear programming model for this problem.

Solution:

Let x_1, x_2 be the number of window frames to be produced of wood framed and aluminium framed respectively.

Total profit would be $60x_1 + 30x_2$

Thus,

$$Z_{max} = 60x_1 + 30x_2 \text{ is the objective function.}$$

It is given that

Doug makes 6 wood frames per day and Linda makes 4 aluminum frames per day.

Hence, $x_1 \leq 6,\ x_2 \leq 4$

Also, it is given that Bob forms and cuts the glass and can make 48 sq. ft of glass per day. Each wood framed window requires 6 sq. ft of glass and each aluminium framed window requires 8 sq. ft of glass.

Hence,

$$6x_1 + 8x_2 \leq 48$$

Thus,

$$Z_{max} = 60x_1 + 30x_2$$

Such that $x_1 \leq 6, \quad x_2 \leq 4$

$$6x_1 + 8x_2 \leq 48 \text{ and } x_1, x_2 \geq 0.$$

4. *A manufacturer produces three models (say, keyboards of computer) I, II, III of certain product using raw materials A and B, the following table gives data of the problem*

Raw material	*Requirements per unit*			*Availability*
	I	*II*	*III*	
A	*2*	*3*	*5*	*4000*
B	*4*	*2*	*7*	*6000*
Minimum demand	*200*	*200*	*150*	-
Profit per unit (Rs.)	*30*	*20*	*50*	-

Formulate the problem as a linear program model.

Solution:

Let x_1, x_2, x_3 be the numbers of models of I, II and III respectively.

The objective function $Z_{max} = 30x_1 + 20x_2 + 50x_3$

Subject to constraints

$2x_1 + 3x_2 + 5x_3 \leq 4000$ (availability of raw material 'A')

$4x_1 + 2x_2 + 7x_3 \leq 6000$ (availability of raw material 'B')

$x_1 \geq 200$ (minimum demand of model I)

$x_2 \geq 200$ (minimum demand of model II)

$x_3 \geq 150$ (minimum demand of model III) and $x_1, x_2, x_3 \geq 0$

Note: *If the requirement/demand is mentioned as minimum, then the constrain should be of type* $\geq$ *and when it is mentioned as maximum, then it should be of* $\leq$ *type.*

5. *A chemist requires 10, 12, and 12 units of chemicals X, Y and Z respectively for his garden. A liquid product contains 5, 2 and 1 units of X, Y and Z respectively. A dry product contains 1, 2 and 4 units of X, Y and Z per carton. If the liquid product sells for ₹ 30/– per jar and the dry product sells for ₹ 20/– per carton. Formulate the problem as an LPP.*

Solution

Let x_1, x_2 be the number of units of liquid and dry products respectively.

Since the cost for the product is given,

Minimum Z or $Z_{min} = 3x_1 + 2x_2$ is the objective function subject to,

$5x_1 + x_2 \geq 10$ (minimum requirement of chemical 'X')

$2x_1 + 2x_2 \geq 12$ (minimum requirement of chemical 'Y')

$x_1 + 4x_2 \geq 12$ (minimum requirement of chemical 'Z')

The inequality used is $\geq$ as it is mentioned that the person requires minimum of 10, 12 and 12 units of chemicals. In other words minimum (not less than) 10, 12 and 12 units of chemicals are required

$x_1, x_2 \geq 0$ (Non–negativity constraint)

Note: *If the objective is to minimize, (such as cost, time or distance) then it will be Z_{min}. On the other hand, if the objective is to maximize, (such as profit or sales) then it will be Z_{max}*

6. *Food X contains 6 units of Vitamin A/gram and 7 units of Vitamin B/gram and costs 20 paise / gram. Food Y contains 8 units of Vitamin A/gram and 12 Units of Vitamin B/gram and costs 30 paise / gram. The daily minimum requirement of Vitamin A and B are 100 units and 120 units respectively. Formulate the problem for optimum product mix.*

Solution:

Let x_1, x_2 be the number of units of food X and food Y respectively.

The objective function is

$Z_{min} = 20x_1 + 30x_2$ (cost)

Subject to the constraints

$6x_1 + 8x_2 \geq 100$ (as daily minimum requirement of vitamin 'A' is 100)

$7x_1 + 12x_2 \geq 120$ (as daily minimum requirement of vitamin 'B' is 120)

$x_1, x_2 \geq 0.$

7. *Old hens can be bought at ₹50/– each but young ones cost ₹100/– each. The old hens lay 3 eggs / week and young hens 5 eggs/week. Each egg costs ₹2/–. A hen costs ₹5/– per week to feed. If a person has only ₹2,000/– to spend for hens, formulate the problem to decide how many of each kind of hen should he buy? Assume that he cannot house more than 40 hens.*

Solution:

Let x_1, x_2 be the number of old and young hens to be purchased.

Number of eggs laid by old hens = 3, number of eggs laid by young hens = 5.

Total income from the eggs

$(3x_1 + 5x_2) \times 2 = 6x_1 + 10x_2$ **...(i)**

(Number of eggs) × selling price

Feeding cost $(x_1 + x_2) \times 5 = 5x_1 + 5x_2$ **...(ii)**

Profit = Income – feeding cost

Therefore, profit = $x_1 + 5x_2$ – is to be maximized

Thus, $Z_{max} = x_1 + 5x_2$ – is the objective function

Subject to the constraints,

$50x_1 + 100x_2 \leq 2000$ – (Budget constraint)

$x_1 + x_2 \leq 40$ – (Housing capacity constraint)

$x_1, x_2 \geq 0$ – (Non negative constraint)

8 *Old machines can be bought at ₹ 2 lakhs each and new machines at ₹ 5 lakhs each. The old machines produce 3 components / week, while new machines produce 5 components / week, each component being worth ₹ 30000. A machine (new or old) costs ₹ 1 lakh / week to maintain. The company has only ₹ 80 lakhs to spend on the machines. How many of each kind should the company buy to get a profit of more than ₹ 6 lakhs / week? Assume that the company cannot house more than 20 machines. Formulate the problem and solve it graphically.*

Solution:

Let x_1, x_2 be the number of old and new machines to purchase. It is given that an old machine can produce 3 components / week, a new machine 5 components / week.

∴ Total components produced / week = $3x^1 + 5x_2$

Also it is given that the worth of each component is ₹ 30,000 = ₹ 0.3 lakh.

Hence, the total gain /income = $0.3 (3x_1 + 5x_2)$ (in lakhs of rupees)

It is also given that the machine (new or old) costs ₹ 1 lakh / week for maintenance. So, the expenditure = $1(x_1 + x_2)$ (in lakhs of rupees)

Profit = Total gain (Income) – Expenditure

$= 0.3 (x_1 + 5x_2) - (x_1 + x_2)$

$Z_{max} = -0.1x_1 + 0.5x_2$ (i) (Objective function)

Constraints:

The buying cost of an old machine is ₹ 2 lakh and of new machine is ₹ 5 lakh.

Total cost = $2x_1 + 5x_2$

It is given that the company has only ₹ 80 lakh to spend,

Hence, $2x_1 + 5x_2 \leq 80$

Further it is given that the company cannot house more than 20 machines i.e. $x_1 + xz_2 \leq 20$ and the company should get a profit of more than ₹ 6 lakh / week. We already know that the profit equation as $0.1\ x_2 + 0.5\ x_2$

Thus, $-0.1\ x_1 + 0.5\ x_2 \geq 6$

The LPP is,

$Z_{max} = -0.1\ x_1 + 0.5\ x_2$

$2x_1 + 5x_2 \leq 80$

$x_1 + x_2 \leq 20$

$-0.1\ x_1 + 0.5\ x_2 \geq 6$

$x_1, x_2 \geq 0$

9. *A farmer has to plant two kinds of trees P and Q in a land of 4400 m^2 area. Each P tree requires at least 25 m^2 and Q tree requires at least 40 m^2 of land. The annual water requirements of P tree is 30 units and of Q tree is 15 units per tree, while at most 3300 units of water is available. It is also estimated that the ratio of the number Q trees to the number of P trees should not be less than 6/19 and should not be more than 17/8. The return per tree from P is expected to be one and half times as much as from Q tree.*

Formulate the problem as a LP model.

Solution:

Let x_1, x_2 be the number of type 'P' trees and 'Q' type trees respectively.

Note:

For the convenience, the data given may be tabulated as

	Type P	Type Q	Availability
Profit	1.5	1	–
Area	25	40	4400 (m^2)
Water	30	15	3300 (units)

Now, it will be easy to formulate as,

$Z_{max} = 1.5x_1 + 1x_2$ (as the profit on type P tree is 1.5 times more than on type Q tree's profit)

Subject to,

$25x_1 + 40x_2 \leq 4{,}400$ (Area constraint)

$30x_1 + 15\ x_2 \leq 3{,}300$ (Water constraint)

It is given that ratio of number of trees of type 'Q' to that of type 'P' should be in the range of 6/19 to 17/8 (not less than 6/19 but not more than 17/8) i.e. $\frac{17}{8} \leq \frac{x_2}{x_1} \geq \frac{6}{19}$ or

$\frac{x_2}{x_1} \geq \frac{6}{19}$, $\frac{x_2}{x_1} \leq \frac{17}{18}$ and $x_1, x_2 \geq 0$

10. *A firm can produce 3 types of body sweaters say A, B and C. Three kinds of wool are required for it, say red wool, green wool and blue wool. One unit of type 'A' sweater needs two yards of red wool and three yards of blue wool, one unit of type B sweater needs 3 yards of red wool, 2 yards of green wool and 2 yards of blue wool. One unit of type 'C' sweater needs 5 units of green wool and 4 yards of blue wool. The firm has only a stock of 80 yards of red wool, 100 yards of green wool and 150 yards of blue wool. It is assumed that the income obtained from each unit of type 'A' sweater is ₹ 30, type 'B' sweater is Rs. 50 and type 'C' sweater is Rs. 40. Formulate this problem as LPP.*

Solution:

For the convenience, the given data can be represented in the tabular form as shown.

Type of wool	*Type of sweater*			*Quantity of wool available*
	A	*B*	*C*	
Red	*2*	*3*	*0*	*80*
Green	*0*	*2*	*5*	*100*
Blue	*3*	*2*	*4*	*150*
Income (Rs.)	*30*	*50*	*40*	–

Let x_1, x_2 and x_3 be the number of sweaters of type 'A', type 'B' and type 'C' to be produced.

It is given that Rs. 30, Rs. 50 and Rs. 40 as the income per one unit of type 'A', type 'B' and type 'C' sweaters respectively.

Hence, $\mathbf{Z_{max} = 30x_1 + 50x_2 + 40x_3}$ is the objective function.

Constraints:

As per the given data we have constraints on availability of the red, green and blue wool.

Red wool:

Since 2 yards of red wool is required for each of type 'A' sweater, $2x_1$, yards of red wool will be required for type 'A' sweaters.

Type 'B' sweater requires $3x_2$ yards of red wool and type 'C' doesnot require red wool.

Hence, the total quantity of red wool becomes (for all the 3 types of sweaters).

$2x_1 + 3x_2 + 0x_3$

as it is given that not more than 80 yards of red wool is available;

$2x_1 + 3x_2 \leq 80$

Similarly; for green wool, we get, $2x_1 + 5x_3 \leq 100$ and

for Blue wool $3x_1 + 2x_2 + 4x_3 \leq 150$

x_1, x_2 and $x_3 \geq 0$

***Note:** Yard is a unit of length 1 yard = 3 ft. = 0.9144 m*

11. *ABC firm manufactures three products P_1, P_2 and P_3. The profits are ₹30, ₹20 and ₹40 respectively. The firm has two machines M_1 and M_2 and requires processing time in minutes for each machine on each product and total machine available minutes on each machine are given below.*

Machine	*Machine minutes required*			*Total Machine minutes available*
	P_1	P_2	P_3	
M_1	4	3	5	2000
M_2	2	2	4	2500

The firm must manufacture at least 100 P_1's and 200 P_2's and 50 P_3's but not more than 150 P_1's. Set up an LP model to solve by simplex method.

Solution:

Let x_1, x_2 and x_3 be the number of products of P_1, P_2 and P_3 type respectively. Then the objective function $Z_{max} = 30x_1 + 20x_2 + 40x_3$.

Subject to,

$4x_1 + 3x_2 + 5x_3 \leq 2000$	(i)	MachineM_1 constraint
$2x_1 + 2x_2 + 4x_3 \leq 2500$	(ii)	Machine M_2 constraint
$x_1 \geq 100$	(iii)	Min.requirement of P_1
$x_2 \geq 200$	(iv)	Min.requirement of P_2
$x_3 \geq 50$	(v)	Min.requirement of P_3
$x_1 \leq 150$	(vi)	Max.requirement of P_1

$x_1, x_2, x_3 \geq 0$(non–negative constraint)

12. *A farmer has 100 acre land. He can sell all the tomatoes, lettuce or radishes he can raise. The price he can obtain is ₹10/- per kg for tomatoes, ₹7/- a head for lettuce and ₹10/- per kg for radishes. The average yield per acre is 2,000 kg of tomatoes, 3000 heads of lettuce and 1,000 kg radishes. Labor required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes and 6 man-days for lettuce. A total of 400 man-days of labour are available at ₹100/- per man day, Formulate this problem as LPP to maximize the farmer's total profit.*

Solution:

Let x_1, x_2 and x_3 be the number of acres of land in which tomatoes, lettuce and radishes are grown respectively in order to maximize the profit.

So, the farmer can produce / grow

$2000x_1$ kgs of tomato

$3000x_2$ heads of lettuce

$1000x_3$ kgs of radish

Hence, the total sales income of the farmer is

$2000x_1 \times 10 + 3000x_2 \times 7 + 1000x_3 \times 10$

Expenditure on the labour is, $100\,(5x_1 + 6x_2 + 5x_3)$

Therefore, the farmer's net profit = (total sales income) – (total expenditure)

$Z = (20{,}000x_1 + 21{,}000x_2 + 10{,}000) - (500x_1 + 600x_2 + 500x_3)$

$Z = 19{,}500x_1 + 20{,}400x_2 + 9500x_3$

Hence, the objective function is,

$Z_{max} = 19{,}500x_1 + 20{,}400x_2 + 9{,}500\,x_3$

Subject to,

$x_1 + x_2 + x_3 \leq 100$ (land costraint)

$5x_1 + 6x_2 + 5x_3 \leq 400$ (man days constraint)

$x_1, x_2, x_3 \geq 0$

13. *A toy company manufactures two types of dolls, a basic version doll A and a deluxe version doll B. Each doll of type B takes twice as long to produce as one of type A and the company would have time to make maximum of 2,000 dolls per day. The supply of plastic is sufficient to produce 1,500 dolls per day (Both A and B combined). The deluxe version requires a fancy dress of which there are only 600 per day available. If the company makes a profit of ₹10/- and ₹18/- per doll on doll A and B respectively, then how many of each doll should be produced per day in order to maximize the total profit. Formulate the problem as LPP.*

Solution:

Let x_1, x_2 be the number of dolls produced per day of type A and B respectively. Let the doll A require 't' hours so that the doll B requires 2t hours.

Therefore, the total time to manufacture x_1 and x_2 dolls should not exceed 2,000t hours that is $tx_1 + 2tx_2 \leq 2000t$.

The LPP is,

$Z_{max} = 10x_1 + 18x_2$

Subject to the constraints

$x_1 + 2x_2 \leq 2000$ (Time constraint)

$x_1 + x_2 \leq 1{,}500$ (Plastic supply constraint)

$x_2 \leq 600$ (Fancy dress constraint)

$x_1, x_2 \geq 0$.

14. *The standard weight of a special purpose brick is 5 kg and it contains two ingredients B_1 and B_2. B_1 costs ₹5/– per kg and B_2 costs ₹8/– per kg. Strength considerations dictate that the brick contains not more than 4 kg of B_1 and a minimum of 2 kg of B_2, since the demand for the product is likely to be related to the price of the brick. Formulate the above problem as a L.P model.*

Solution:

Let x_1 and x_2 be the ingredients of B_1 and B_2 in the brick respectively.

Then, $Z_{min} = 5x_1 + 8x_2$ is the objective function subject to,

$x_1 \leq 4$ $x_2 \geq 2$	Strength constraints
$x_1 + x_2 = 5$	Standard weight constraint
$x_1, x_2 \geq 0$	Non negative constraint

15. *A marketing manager wishes to allocate his annual advertising budget of ₹ 20,000 in two media groups M and N. The unit cost of the message in the media 'M' is ₹ 200 and 'N' is ₹ 300. The media M is monthly magazine and not more than two insertions are desired in one issue. At least five messages should appear in the media N. The expected effective audience per unit message for Media M is 4,000 and for N is 5,000. Formulate the problem as Linear Programming problem.*

Solution:

Let x_1, x_2 be the number of times of advertising in the media M and N respectively.

Then, $Z_{max} = 4000x_1 + 5000x_2$

Subject to $200x_1 + 300x_2 \leq 20{,}000$ (Budget constraint)

$x_1 \leq 2$ (not more than 2 insertions)

$x_2 \geq 5$ (at least 5 messages to insert)

$x_1, x_2 \geq 0$ (non – negative constraint)

16. *The Apex television company has to decide on the number of 27 inch and 20 inch sets to be produced at one of its factories. Market research indicates that at most 40 of the 27-inch sets and 10 of 20 inch sets can be sold per month. The maximum number of work hours available is 500 per month. A 27 inch set requires 20 work hours and 20-inch set requires 10 work hours. Each 27 inch set sold produces a profit of $120 and each 20 inch produces a profit of $80. A wholesaler agreed to purchase all the television sets produced, if the numbers do not exceed the maxima indicated by market research. Formulate a linear programming model for the problem.*

Solution:

Let x_1, x_2 be the number of 27 inch and 20 inch sets respectively to be produced.

The objective function is

$Z_{max} = 120x_1 + 80x_2$

Subject to

$x_1 \leq 40$

$x_2 \leq 10$ (at most implies not more than the specified quantity)

$20x_1 + 10x_2 \leq 500$ (work hours constraint)

17. *The world light company produces two light fixtures requiring both metal frame parts and electrical components. The management wishes to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of $1 and each unit of product 2, upto 60 units, gives a profit of $2, any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out.*

Formulate a LPP model for this problem

Solution:

Let x_1, x_2 be the number of units of product 1 and product 2 respectively.

The objective function is, $Z_{max} = 1x_1 + 2x_2$

Subject to,

$1\ x_1 + 3\ x_2 \leq 200$ (Availability of frame parts)

$2\ x_1 + 2\ x_2 \leq 300$ (Availability of electrical components)

$x_2 \leq 60$ (Restriction on quantity of product 2)

$x_1, x_2 \geq 0$

18. *An agricultural Research institute suggested to a farmer to spread out at least 4800 kg of a special Phosphate fertilizer and not less than 7200 kg of a special nitrogen fertilizer to raise productivity of crops in his fields. There are two sources for obtaining these – mixtures A and B. Both of these are available in bags weighing 100 kg each and they cost Rs. 40 and Rs. 24 respectively. Mixture A contains phosphate and nitrogen equivalent of 20 kg and 80 kg respectively, while mixture B contains these ingredients equivalent of 50 Kg each. Write this as a linear program to determine how many bags of each type the farmer should buy in order to obtain the required fertilizer at minimum cost.*

Solution:

To formulate the given problem as LPP we need to define the objective function, constraints.

i) ***Objective Function***

Let x_1, x_2 be the number of bags of mixtures A and B respectively.

It is given that, cost per one bag of mixture A is Rs. 40. Therefore, cost per x_1 bags is $40x_1$

Similarly, as cost per one bag of mixture B is Rs. 24, cost per x_2 bags is $24\ x_2$

Hence, the objective function is

$Z_{min} = 40\ x_1 + 24\ x_2$

(As it is cost, the objective is to minimize it)

ii) ***Constraints***

There are two constraints namely a minimum of 4,800 Kg of phosphate and 7,200 Kg of nitrogen ingredients are required.

It is given that each bag of mixture A contains 20 Kg and each bag of mixture B contains 50 Kg of phosphate. The total Phosphate available from bag of mixture A is $20\ x_1$ and from mixture B it is $50\ x_2$.

Hence, the total availability of the phosphate from A and B is $20x_1 + 50x_2$

Thus, the phosphate requirement (constraint) can be expressed as

$20\ x_1 + 50\ x_2 \geq 4{,}800$.

Similarly, with the given data, the nitrogen requirement would be written as

$80\ x_1 + 50\ x_2 \geq 7{,}200$.

Note: *Whenever the constraint says minimum requirement it should be at least that much or greater. Hence, mathematically ≥ is used.*

Non – negativity Constraint

The decision variables, representing the number of bags of mixtures A and B, would be non – negative.

Thus, $x_1 \geq 0,\ x_2 \geq 0$

The linear programming can now be expressed as,

$Z_{min} = 40\,x_1 + 24\,x_2$

Subject to

$20\,x_1 + 50\,x_2 \geq 4{,}800$ (Phosphate requirement)

$80\,x_1 + 50\,x_2 \geq 7{,}200$ (Nitrogen requirement) and

$x_1,\ x_2 \geq 0$ (Non – negativity constraint)

19. *A company has two bottling plants one located at Bangalore and the other located at Mysore. Each plant produces 3 brands of soft drinks Thums Up, Limka and Coke say A, B and C respectively. Bangalore plant can produce 1500, 3000 and 2000 bottles of A, B and C in a day respectively while the capacity of Mysore plant is 1500, 1000, 5000 bottles of A, B and C per day respectively. Market survey indicates that during the month of April there will be a minimum demand of 20,000 bottles of A, 40,000 of B and 44,000 of C. The operating cost / day for Mysore plant is Rs.4000/- and for Bangalore plant is Rs.6000/-. For how many days should the plant run in April so as to minimize production cost, while still meeting the demands (only formulate).*

Solution:

Let x_1, x_2 be the number of days that Bangalore and Mysore plants are proposed to operate.

Then the objective function is

Min. $Z = 6000x_1 + 4000x_2$

Subject to the constraints

$1500x_1 + 1500x_2 \geq 20000$ (Requirement of type 'A' bottles)

$3000x_1 + 1000x_2 \geq 40000$ (Requirement of type 'B' bottles)

$2000x_1 + 5000\,x_2 \geq 44000$ (Requirement of type 'C' bottles)

and $x_1, x_2 \geq 0$

20. *The manager of an oil refinery has to decide upon the optimal mix of two possible blending process of which the inputs and outputs per production run are as follows:*

Input			*Output*	
Process	*Crude A*	*Crude B*	*Gasoline X*	*Gasoline Y*
1	*5*	*3*	*5*	*8*
2	*4*	*5*	*4*	*4*

The maximum amounts available of crude A and B are 200 and 150 units respectively. Market requirement show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profit per production run from process 1 and process 2 are Rs.3 and Rs. 4 respectively. Formulate the problem as LP model.

Solution:

Let x_1, x_2 be the number of runs of process 1 and 2 respectively

$Z_{max} = 3x_1+4x_2$

Subject to

$5x_1+4x_2 \leq 200$ (crude 'A' Constraint)

$3x_1+5x_2 \leq 150$ (Crude 'B' Constraint)

$5x_1+4x_2 \geq 100$ (Gasoline'X'Constraint)

$8x_1+4x_2 \geq 80$ (Gasoline'Y'Constraint)

$x_1, x_2 \geq 0$

Students are advised to declare the decision variables clearly and then frame the objective function, constraints after understanding / analysis of the problem.

1.15 SOLUTION OF AN LPP BY GRAPHICAL METHOD

So far, we have discussed how to formulate a linear programming problem. It can be solved to obtain the decision variables declared in the objective function subjected to the given constrains. Linear programming problems involving two decision variables (say x_1, x_2) can easily be solved by graphical method, which provides a pictorial representation of the solution. When there are more than two decision variables involved in the linear programming problem, then an iterative method known as simplex method is used to solve the problem. Graphical method is discussed in this chapter and simplex method, its variants are discussed in subsequent chapters.

Steps in Graphical Method (Working Procedure)

i. *Formulate the given problem as LPP*

ii. *Draw a graph with one variable on the horizantal axis and one on the vertical axis.*

iii. *Plot each of the constraints as if they were equalities or equations.*

iv. *Identify the feasible region (solution space) that is the area that satisfies all the constraints.*

v. *Name the intersections of the constraints on the perimeter of the feasible region and get their co–ordinates*

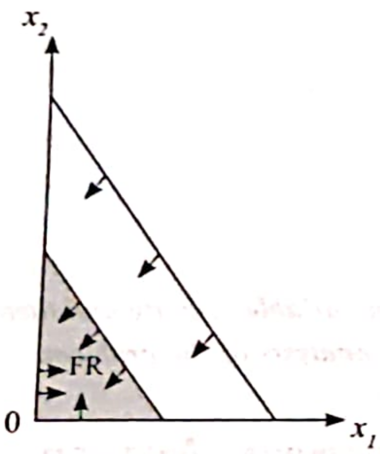
vi. *Substitute each of the co–ordinates into the objective function and solve for Z*

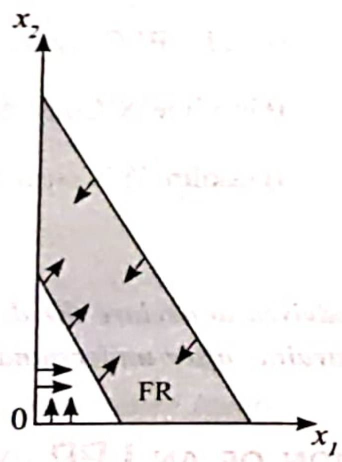
vii. *Select the solution that optimizes Z (based on the objective) that is obtain* Z_{min} *or* Z_{max}

Guidelines in identifying feasible region

Plot each constraint on the graph by treating it as a linear equation. Based on the constraint, whether it is ≥ or ≤, represent the arrows upwards or downwards respectively.

The inequality conditions and non negativity condition can only be satisfied by the shaded area (*feasible region*) as shown below.





The shaded portion is Feasible Region (FR), a common area satisfying the given constraints.

Method of Corner Points: The method of finding the optimal solution to a linear programming problem by testing/ calculating the profit or cost at each corner point of the feasible region.

Let us understand the procedure of obtaining the solution of an LPP by graphical method by the following examples.

Important Definitions:

Solution: The set of values of decision variables Xj (j = 1, 2, 3...n) which satisfy the constraints of the LP problem is said to that LPP's solution.

Feasible solution: The set of values of decision variables Xj (j = 1, 2, 3...n) which satisfy all the constraints and non negativity conditions of an LPP simultaneously is said to constitute the feasible solution.

Basic Feasible Solution: A feasible solution solution of an LPP with all the basic variables as non-negative values.

Feasible Region: Feasible solution: - The set of values of decision variables Xj (j = 1, 2, 3...n) which satisfy all the constraints and non negativity conditions of an LPP simultaneously is said to constitute the feasible solution.

Optimal Solution: A basic feasible solution which optimizes the objective function of the given LPP is called an optimal solution.

Note: *An optimal solution should necessarily be feasible but a feasible solution may or may not be optimal.*

21. *Solve the following LPP by graphical method.*

$Z_{max} = 3x_1 + 4x_2$

Subject to $x_1 + x_2 \leq 450$, $2x_1 + x_2 \leq 600$ *and* $x_1, x_2 \geq 0$

Solution:

Converting the inequalities into equations we get,

$x_1 + x_2 = 450$, $2x_1 + x_2 = 600$

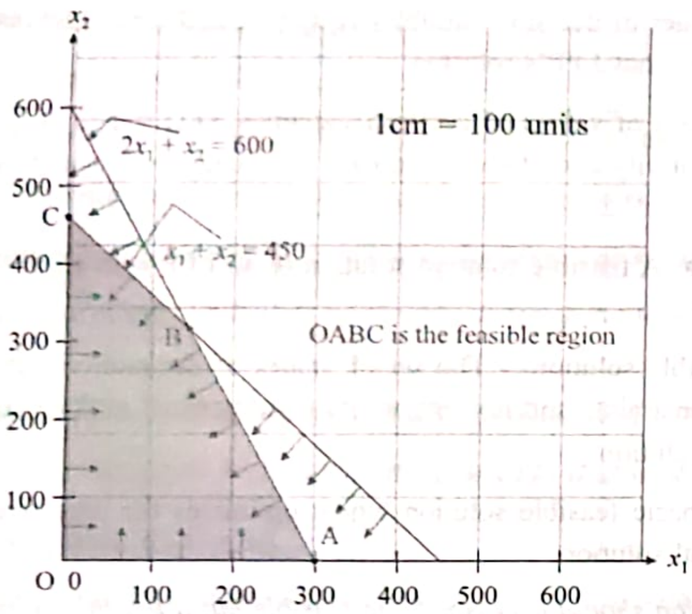
$x_1 + x_2 = 450$ passes through (0, 450); (450,0)

[Assuming $x_1=0$, $x_2 = 450$ and $x_2 = 0$, $x_1 = 450$].

Similarly, $2x_1 + x_2 = 600$ passes through, (0, 600) and (300, 0) plot the co–ordinates on the graph sheet

Mathematically / graphically any set of values of x_1 and x_2 lying on or below the equation or line will satisfy the constraint $\leq$, in other words represent the arrows downwards if the constrain is $\leq$ and upwards if the constraint is $\geq$.

Now, the common area shared by considering all these constraints is the feasible region or the solution space (OABC) that is the most common area between upward and downward arrows in the graph.



Using the method of corner points or vertices (which are the bounded points of the region), we can find the value of Z.

$Z_{max} = 3x_1 + 4x_1$

$Z_O = 3(0) + 4\ (0) = 0$

$Z_A = 3(300) + 4\ (0) = 900$

$Z_B = 3(150) + 4\ (300) = 450+1200 = 1650$

$Z_C = 3(0) + 4\ (450) = 1800$

Z_{max} occurs at C, the value = 1800.

Hence, the corresponding co–ordinates of x_1, x_2 are

$x_1 = 0,\ x_2 = 450$

Observation:

Substituting the obtained values of x_1 and x_2 it can be observed that. the constraints are satisfied.

i.e. with $x_1 = 0$, $x_2 = 450$

$x_1 + x_2 \leq 450$, $2x_1 + x_2 \leq 60$

22. ***A firm manufactures two types of products A and B and sells them at a profit of Rs. 20-- on type 'A' and Rs. 30/- on type 'B'. Each product is processed on two machines G and H. Type 'A' requires one minute of processing type on G and two minutes on H. Type B requires one minute on G and one minute on H. The machine G is available for not more than 6 hours 40 minutes while H is available for 10 hours during any working day. How many items of type 'A' and type 'B' should be produced so that the total profit is maximum?***

(i) use mathematical formulation to the LPP.

(ii) use graphical method to solve the problem.

Solution:

The given data can be conveniently represented by a table as given below.

Machine	*Processing time*		*Available time (Minutes)*
	Type 'A'	*Type 'B'*	
G	*1*	*1*	*400*
H	*2*	*1*	*600*
Profit (Rs.)	20	30	–

→ 40 + 6 × 6 = 600

→

6 hours 40 minutes = 400 minutes.

Let x_1, x_2 be the number of units of Type 'A' and Type 'B' products respectively.

It is given that profit on each unit of type 'A' and type 'B' products as Rs. 20 and Rs. 30 respectively.

$Z_{max} = 20x_1 + 30x_2$

Constraints:

Since machine G takes1 minute time on type A product, 1 minute on type B product and as its availability is 400 minutes.

$x_1 + x_2 \leq 400$

Similarly, for machine 'H' the constraint is

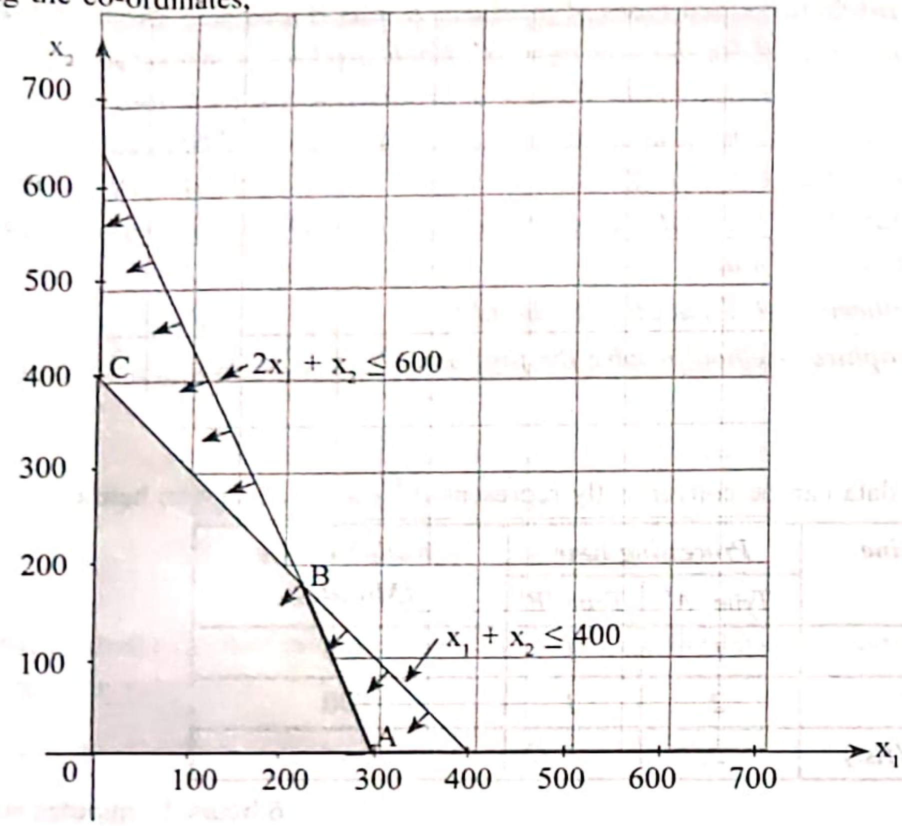
$2x_1 + x_2 \leq 600$ and $x_1, x_2 \geq 0$

Solution:

Converting the in-equalities (constraints) into equations we get,

$x_1 + x_2 = 400$	*passes through*	*(0, 400) ; (400, 0)*
$2x_1 + x_2 = 600$	*passes through*	*(0, 600) ; (300, 0)*

Plotting the co-ordinates,



Scale: 1cm = 100 units

OABC is the Feasible region (the common region satisfying the constraints)

The value of Z at each corner of the Feasible Region:

$Z = 2x_1 + 3x_2$

$Z_{(0)} = 20(0) + 30(0) = 0$

$Z_{(A)} = 20(300) + 30(0) = 60000$

$Z_{(B)} = 20(200) + 30(200) = 100000$

$Z_{(C)} = 20(0) + 30(400) = 120000$

Z_{max} occurs at 'C'. Hence, $x_1 = 0$, $x_2 = 400$.

Observation: Substituting $x_1 = 0$, $x_2 = 400$ the constraints are satisfied.

23. ***A retailer deals in two items only, item 'A' and item 'B'. He has Rs. 50,000 to invest and a space to store at most 600 items. An item 'A' costs him Rs. 2500 and 'B' costs Rs. 500. A net profit to him on item 'A' is Rs. 500 and item 'B' is Rs. 150.***

If he can sell all the items he purchased, how should he invest his amount to have maximum profit ?

(i) Give mathematical formulation to the LPP.

(ii) Use graphical method to solve the problem.

Solution:

Let x_1, x_2 be the number of items of type 'A' and type 'B' to be purchased.

It is given that net profit on item A as Rs. 500 and on item 'B' as Rs. 150.

The total profit is, $500x_1 + 150x_2$

Thus, $Z_{max} = 500x_1 + 150x_2$

The person is having two constraints, one on the space and the other on budget.

It is given that, he cannot store more than 60 items i.e.

$$x_1 + x_2 \leq 60$$

and also it is specified that, he has only Rs. 50,000 to invest

hence, $2500x_1 + 500x_2 \leq 50,000$

(each unit of item 'A' costs Rs. 2500 and item 'B' costs Rs. 500)

Thus,

$$\mathbf{Z_{max} = 500x_1 + 150x_2}$$

Subject to $\mathbf{x_1 + x_2 \leq 60}$

$$\mathbf{2500x_1 + 500x_2 \leq 50000 \text{ and } x_1, x_2 \geq 0}$$

Solution:

Converting the inequalities into equations we get,

$x_1 + x_2 = 60$ passes through (0, 60) ; (60, 0)

$2500x_1 + 500x_2 = 50000$

or

$5x_1 + x_2 = 100$ passes through (0, 100) ; (20, 0)

Plotting these co-ordinates we get,

OABC is the Feasible Region (FR)

The value of Z at each of the corners of FR

$Z = 500x_1 + 150x_2$

$Z_{(0)} = 500(0) + 150(0) = 0$

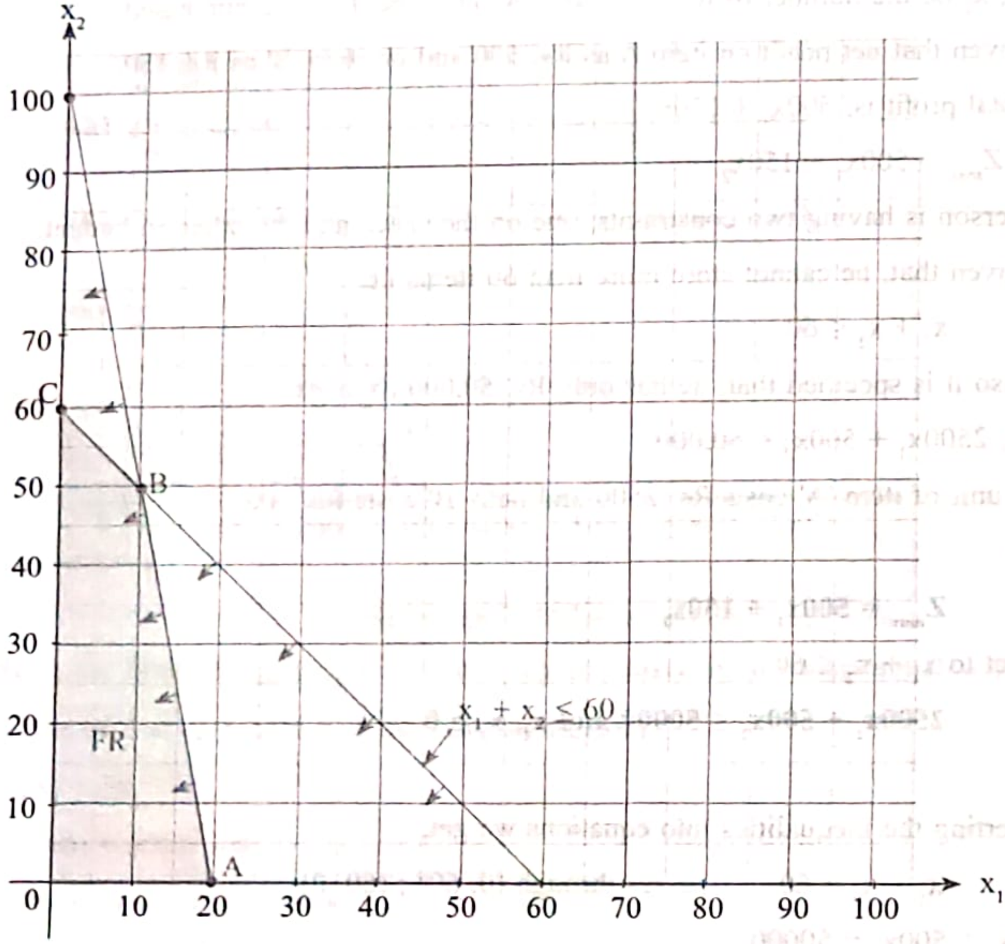
$Z_{(A)} = 500(20) + 150(0) = 10,000$

$Z_{(B)} = 500(10) + 150(50) = 8,000$

$Z_{(C)} = 500(0) + 150(60) = 9,000$

Z_{max} occurs at 'A'.

Hence, $x_1 = 20, x_2 = 0$.



Scale: 1cm = 10 units

24. ***Solve the following LPP by graphical method.***

Minimize $Z = 20x_1 + 10x_2$

Subject to $x_1 + 2x_2 \le 40$

$3x_1 + x_2 \ge 30$

$4x_1 + 3x_2 \ge 60$

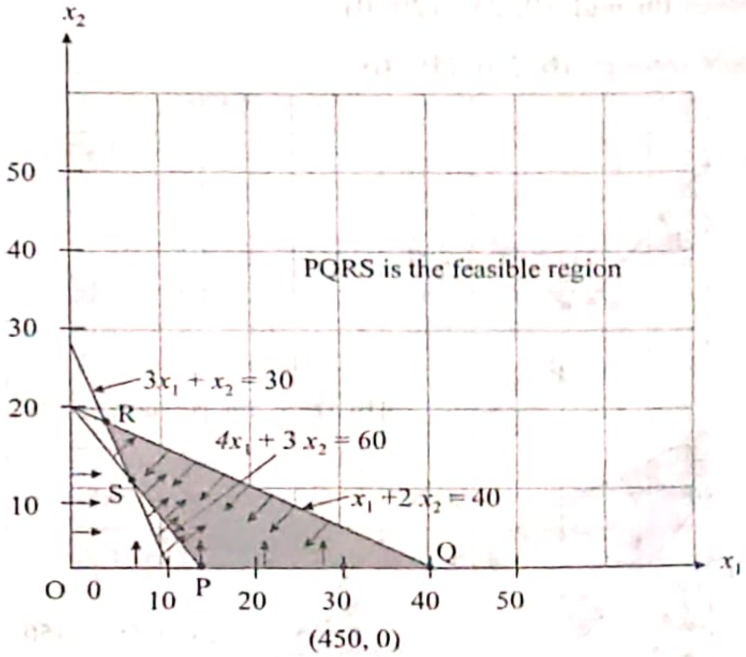
$x_1, x_2 \ge 0.$

Solution:

Replacing all the inequalities as equations we get,

$x_1 + 2x_2 = 40$, $3x_1 + x_2 = 30$, $4x_1 + 3x_2 = 60$. They pass through the co–ordinates (0, 20); (40, 0) (0, 30); (10, 0) (0, 20); (15, 0) respectively.

Plotting the co–ordinates on the graph we get,



Scale: 1cm = 10 units

PQRS is the feasible region. The value of objective function at the corners of the feasible region PQRS is

(Co–ordinates at P, Q, R, S are (15, 0), (40, 0), (4, 18), (6, 12) respectively).

$Z_P = 20(15)+10(0) = 300$

$Z_Q = 20(40) +10(0) = 800$

$Z_R = 20(4) +10(18) = 260$

$Z_S = 20(6) +10(12) = 240$

The minimum value occurs at 'S'. Hence, $Z_{min} = 240$ for which $x_1 = 6$, $x_2 = 12$.

25. ***Using graphical method find $Z_{max} = 3x_1 + 4x_2$ subject to***

$5x_1 + 4x_2 \le 200$ $\quad 3x_1 + 5x_2 \le 150$ $\quad 5x_1 + 4x_2 \ge 100$ $\quad 8x_1 + 4x_2 \ge 80$

and $x_1, x_2 \ge 0$

Solution:

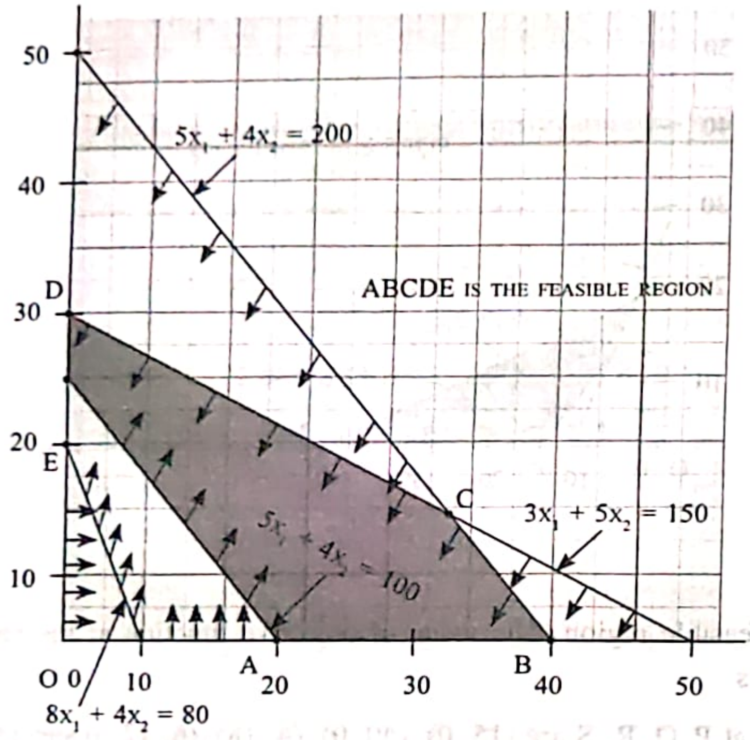
Converting the inequalities (constraints) into equations we get,

$5x_1 + 4x_2 = 200$ passes through (0, 50), (40, 0)

$3x_1 + 5x_2 = 150$ passes through (0, 30), (50, 0)

$5x_1 + 4x_2 = 100$ passes through (0, 25), (20, 0)

$8x_1 + 4x_2 = 80$ passes through (0, 20), (10, 0)



Scale: 1cm = 10 units

Plotting the co–ordinates, we get,

The common feasible region is A B C D E. The value of the objective function at various corners of the feasible region is,

$Z = 3x_1 + 4x_2$

$Z_{(A)} = 3\ (20) + 4\ (0) \qquad = 60$

$Z_{(B)} = 3\ (40) + 4(0) \qquad = 120$

$Z_{(C)} = 3\ (30.8) + 4\ (11.5) \qquad = 138.4$

$Z_{(D)} = 3\ (0) + 4\ (30) \qquad = 120$

$Z_{(E)} = 3\ (0) + 4\ (25) \qquad = 100$

Z_{max} occurs at C and is $\qquad = 138.4$

The corresponding co–ordinates are,

$x_1 = 30.8,\ x_2 = 11.5$

26. *A plant manufactures two products A and B. The profit contribution of each product has been estimated to be Rs. 20 and Rs. 24 for products A and B respectively. Each product passes through two departments of the plant. The time required for each product and the total time available in each department are as follows:*

Department	*Time (hrs) required/unit of*		*Available time (hrs) per month*
	Product - A	*Product - B*	
1	*2*	*3*	*1500*
2	*3*	*2*	*1500*

The plant has to supply the products to market where the maximum demand for product B is 450 units/month. Formulate the problem as an LP model and find graphically, the number of products A and B to maximize the total profit per month.

Solution:

i) Objective Function

Let x_1, x_2 be the number of products of A and B. Profit on one unit of product 'A' is Rs. 20 and product 'B' is Rs. 24. Hence, the total profit is $20x_1 + 24x_2$. Thus, the objective function is,

$Z_{max} = 20x_1 + 24x_2$.

ii) Constraints

$2x_1 + 3x_2 \leq 1500$ (available time in the department 1)

$3x_1 + 2x_2 \leq 1500$ (available time in the department 2)

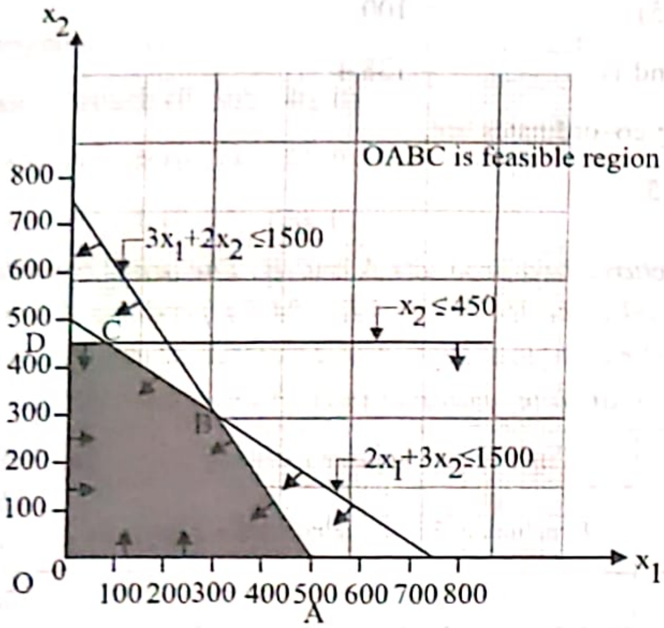
$x_2 \leq 450$ (max.demand for product 'B')

$x_1, x_2 \geq 0$.

Graphical method/solution:

$2x_1 + 3x_2 = 1500$ passes through (0,500); (750,0)

$3x_1 + 2x_2 = 1500$ passes through (0,750); (500,0)



Scale: 1cm = 100 units

OABCD is the feasible region. The values of the objective function at each corner of the feasible region (O, A, B, C and D are the corners of the feasible region).

$Z = 20x_1 + 24x_2$

$Z_{(O)} = 20(0) + 24(0) = 0$

$Z_{(A)} = 20(500) + 24(0) = 10{,}000$

$Z_{(B)} = 20(300) + 24(300) = 13{,}200$

$Z_{(C)} = 20(75) + 24(450) = 12{,}300$

$Z_{(D)} = 20(0) + 24(450) = 10{,}800$

Z_{max} occurs at 'B'. Hence, $x_1 = 300$, $x_2 = 300$.

27. ***A company produces two types of leather belts A and B. Profits on the two types of belts are 40 and 30 rupees per belt respectively. Each belt of type 'A' requires twice as much time as required by belt 'B'. If all the belts were sold of type B, the company could produce 1000 belts per day. The supply of leather is sufficient only for 800 belts per day. Belt 'A' requires a fancy buckle and only 400 fancy buckles are available per day. For belt 'B' only 700 buckles are available per day. How should the company manufacture the two types of belts in order to have maximum overall profit?***

Solution:

Let the company produces two types of leather belts, Type – A and Type – B profits on two types of belts are Rs. 40 and Rs. 30 respectively per bell.

i) ***Objective Function***

$Z_{max} = 40\ x_1 + 30\ x_2$

ii) ***Constraints***

Since belt of type 'A' requires twice as much time as required for a belt of type – B and the company could produce 1000 belts/days.

$2x_1 + x_2 \leq 1000$

But the supply of leather is sufficient only for 800 belts/day.

$x_1 + x_2 \leq 800$

Since, belt 'A' requires a fancy buckle and only 400 fancy buckles are available / day

$x_1 \leq 400$

Similarly, for belt of type A only 700 buckles are available / day.

$x_2 \leq 700$

Non – negative constraint: $x_1 \geq 0, x_2 \geq 0$

Thus, the formulation is

$Z_{max} = 40\ x_1 + 30\ x_2$

Subject to

$2\ x_1 + x_2 \leq 1000$ (1)

$x_1 + x_2 \leq 800$ (2)

$x_1 \leq 400$ (3)

$x_2 \leq 700$ (4)

and $x_1,\ x_2 \geq 0$

Solution by Graphical Method

$2\ x_1 + x_2 = 1000$ passes through (0, 1000); (500, 0) and $x_1 + x_2 = 800$ passes through (0, 800); (800, 0); the co-ordinates of $x_1 \leq 400$ are (400, 0) and of $x_2 \leq 700$ are (0, 700)

Plotting these as straight lines on the graph sheet OABCD is the feasible region.

Substituting the values of x_1, x_2 at the corners of the feasible region we get,

$Z_0 = 40\ (0) + 30(0) = 0$

$Z_A = 40\ (400) + 30(0) = 16{,}000$

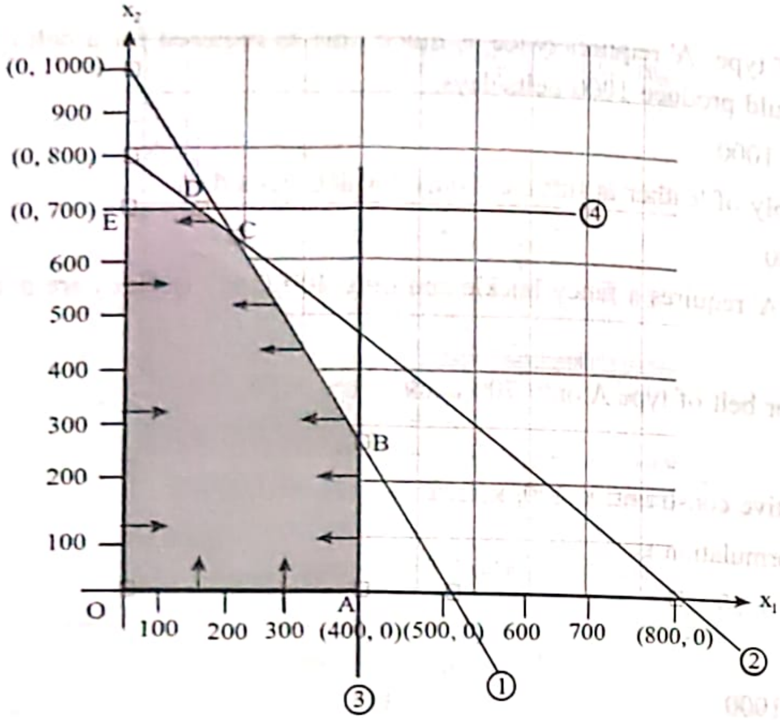
$Z_B = 40\ (400) + 30(200) = 22{,}000$

$Z_C = 40\ (200) + 30(600) = 26{,}000$

$Z_D = 40\ (100) + 30(700) = 25{,}000$

Z_{max} occurs at $x_1 = 200, x_2 = 600$

$Z_{max} = 26{,}000$



Scale: 1cm = 100 units

28. *Use graphical method to solve the following LPP:*

Maximize $Z = x_1 + \frac{x_2}{2}$

Subject to $3x_1 + 2x_2 \le 12$, $5x_1 \le 10$, $x_1 + x_2 \le 18$

$-x_1 + x_2 \ge 4$, *and* $x_1, x_2 \ge 0$

Solution:

Converting the in–equalities as equations we get,

$3x_1 + 2x_2 = 12$; passes through(0,6);(4,0) - - - C_1

$5x_1 = 10$; passes through (2,0) - - - C_2

$x_1 + x_2 = 18$; passes through (0,18);(18,0) - - - C_3

$-x_1 + x_2 = 4$; passes through (0,4);(–4,0) - - - C_4

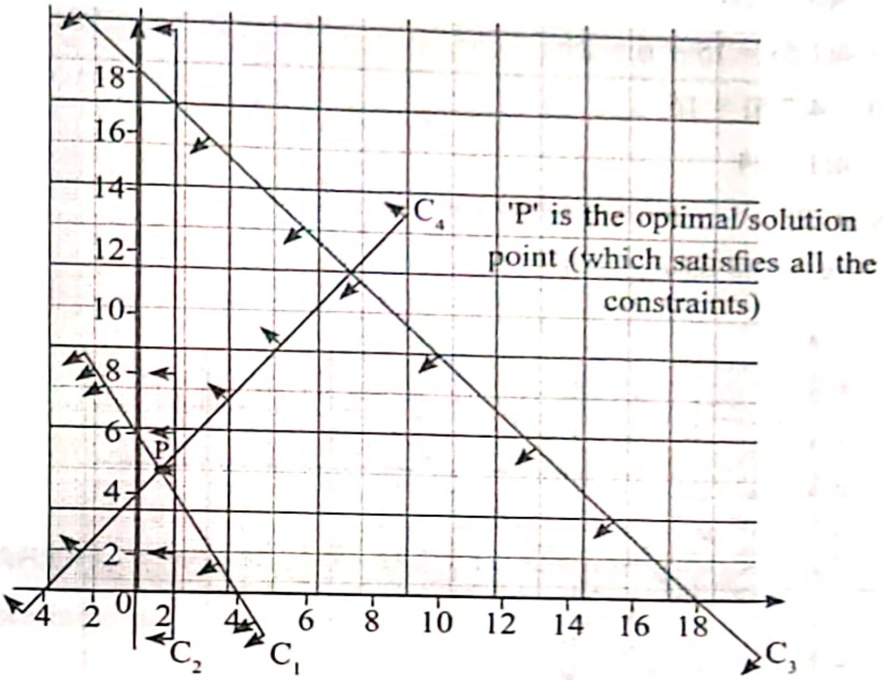
Plotting these on a graph sheet we get,

'P' is the unique point which satisfies all the constraints as shown.

The value of objective function at this point,

$$Z_P = 0.8 + \frac{4.8}{2} = 3.2$$

Hence, $Z_{max} = 3.2$ with $x_1 = 0.8$, $x_2 = 4.8$



Scale: 1cm = 2 units

29. $Z_{max} = 5x_1 + 4x_2$

Subject to $6x_1 + 4x_2 \leq 24$

$x_1 + 2x_2 \leq 6$

$-x_1 + x_2 \leq 1$

Solution:

Converting the in–equalities as equations we get,

$x_1 + 2x_2 = 6$ passes through (0, 3) ; (6, 0)

$-x_1 + x_2 = 1$ passes through (0, 1) ; (–1, 0)

$6x_1 + 4x_2 = 24$ passes through (0, 6) ; (4, 0)

Plotting there co–ordinates

OABCD is the feasible region which satisfies all the constraints (including $x_1, x_2 \geq 0$)

By using the method of corner points, let us find the value of Z at all the consider points of the region.

i.e.,

$Z_O = 5(0) + 4(0) = 0$

$Z_A = 5(4) + 9(0) = 20$

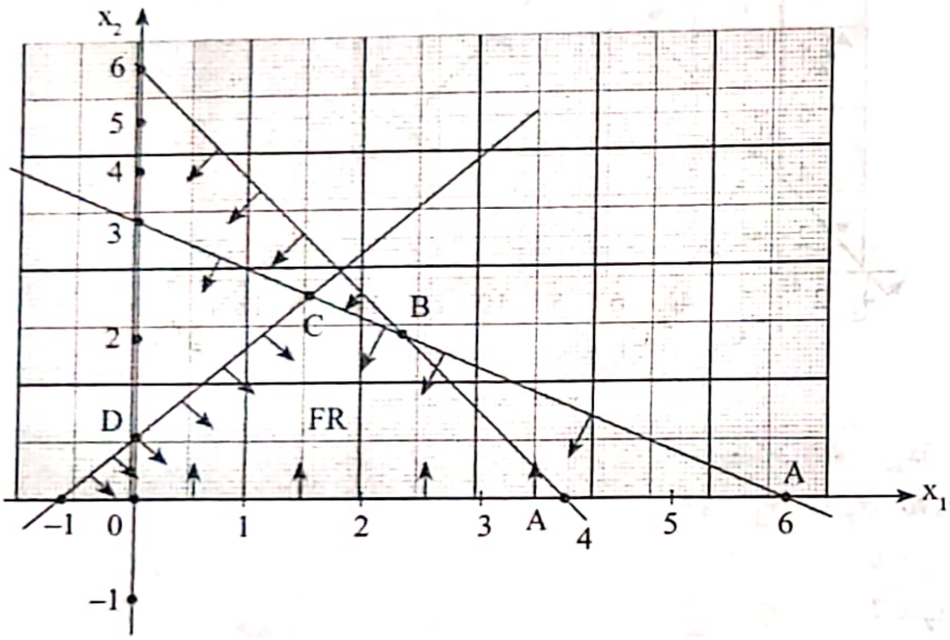
$Z_B = 5(3) + 4(1.5) = 15 + 6 = 21$

$Z_C = 5(4/3) + 4(7/5) = 16$

$Z_D = 5(0) + 4(1) = 4$

Z_{max} occurs at 'B'

Hence, $x_1 = 3$, $x_2 = 1.5$ ad $Z_{max} = 21$



Scale: 1cm = 1 unit

30. *Solve* $Z_{max} = 2x_1 + 3x_2$

Subject to $x_1 + 2x_2 \le 4$

$x_1 + x_2 = 3$

$x_1, x_2 \ge 0$

$x_1 + 2x_2 = 4$ *passes through* $(0, 2); (4,0)$

$x_1 + x_2 = 3$ *passes through* $(0, 3); (3,0)$

Solution:

Plotting these coordinates on the graph sheet we get,

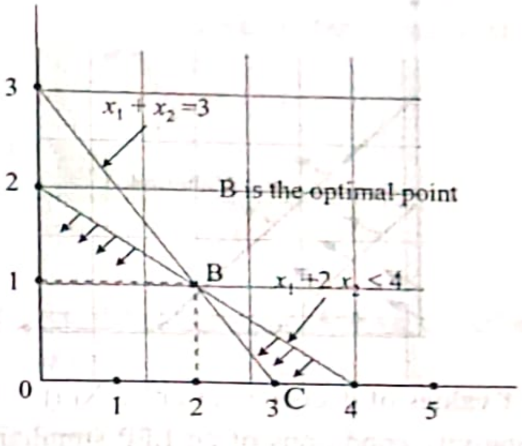
[As $x_1 + x_2 = 3$ is an equation, optimal point is obtained at B]

No common region is found like in previous problems.

$Z_C = 2(3) + 3(0) = 6$

B is the optimal point whose coordinates are (2, 1) and $Z_{max} = 7$

Thus, $x_1 = 2, x_2 = 1$



1.16 Various (Special) Cases in Graphical Method

A Linear–programming problem may be having,

a. ***A unique optimal solution***

b. ***Multiple optimal solutions (alternative optimal solution)***

c. ***An unbounded solution and***

d. ***Infeasible solution***

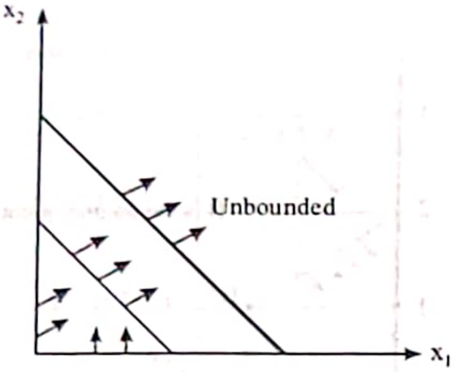
So far we have discussed linear programming problems having unique optimal solution. The following examples will illustrate the linear programming problems having unbounded, alternate optimal solution and no solution cases.

Multiple optional solutions: In usual cases the optimal solution of any linear programming problem occurs at an extreme point of the feasible region and the solution is unique, i.e., no other solution yields the same optimum value of the objective function. However, in certain cases a given LP problem may have more than one optimal solution yielding the same objective functions value. This usually happens whenever the objective function is parallel to a constraint on alternative solution exists.

E.g.:– An objective function $Z_{max} = 4x_1 + 3x_2$ with a constraint $8x_1 + 6x_2 \leq 48$ will have an alternative optimal solution as the objective function is parallel to the constraint (slope of the objective function is same that of the constraint)

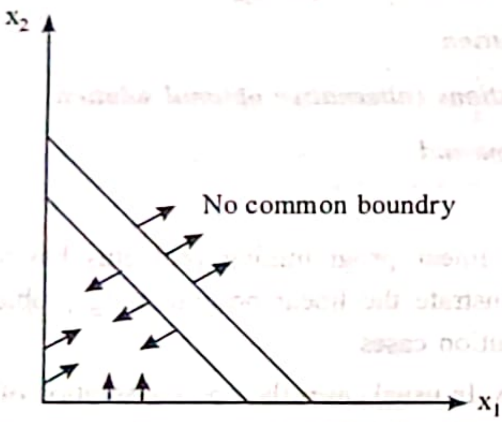
Unbounded Solution: If an LPP has no limit on constraints then it is stated as unbounded LPP. In other words, the common feasible region is not bounded by the given constraints as represented below.

In other words, it is a solution which can increase or decrease the value of the objective of the LPP indefinitely is called an unbounded solution.



Infeasible solution: The set of values of decision variables Xj (j = 1, 2, 3…n) which do not satisfy all the constraints and non negativity conditions of an LPP simultaneously is said to constitute the infeasible solution.

In general, if it is not possible to find a feasible solution satisfying all the constraints, then LPP is said to have an infeasible solution as represented below.



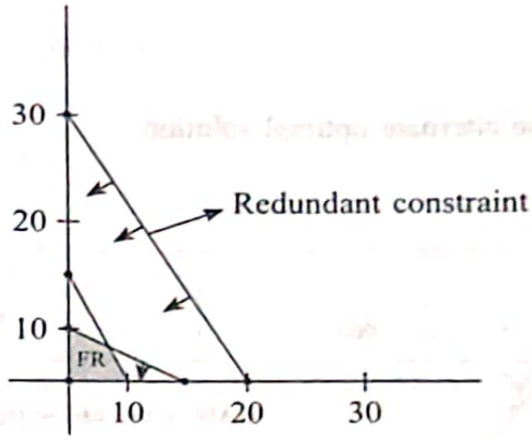
Redundant constraint: It is a system in which deletion of one of the constraints will not affect the feasible region (solution space)

Eg:

$3x_1+2x_2 \leq 30$ passes through (15, 0); (10, 0)

$3x_1+2x_2 \leq 60$ passes through (30, 0); (20, 0)

$2x_1+3x_2 \leq 30$ passes through (10, 0); (15, 0)



From the plot it is evident that even if the constraint shown as redundant constraint is ignored the feasible region will not be affected by it.

The following worked examples illustrate these special cases.

31. *Solve the following LPP by graphical method*

$Z_{max} = 100x_1 + 40x_2$ *subject to*

$5x_1 + 2x_2 \leq 1000,\ 3x_1 + 2x_2 \leq 900$

$x_1 + 2x_2 \leq 500$ *and* $x_1, x_2 \geq 0$

Solution:

Converting the inequalities into equations we get,

$5x_1 + 2x_2 = 100$, $3x_1 + 2x_2 = 900$, $x_1 + 2x_2 = 500$, and they pass through (0, 500); (200, 0); (0, 450); (300, 0) and (0, 250); (500, 0) respectively. Plotting these co–ordinates on the graph sheet, we get,

OABC is the feasible region. The value of objective function

$Z = 100x_1 + 40x_2$ at corner points of the feasible region.

$Z = 100x_1 + 40x_2$

$Z_0 = 100\ (0) + 4\ (0) = 0$

$Z_A = 100\ (200) + 40\ (0) = 20{,}000$

$Z_B = 100\ (125) + 40\ (187.5) = 20{,}000$

$Z_C = 100\ (0) + 40\ (250) = 10{,}000$

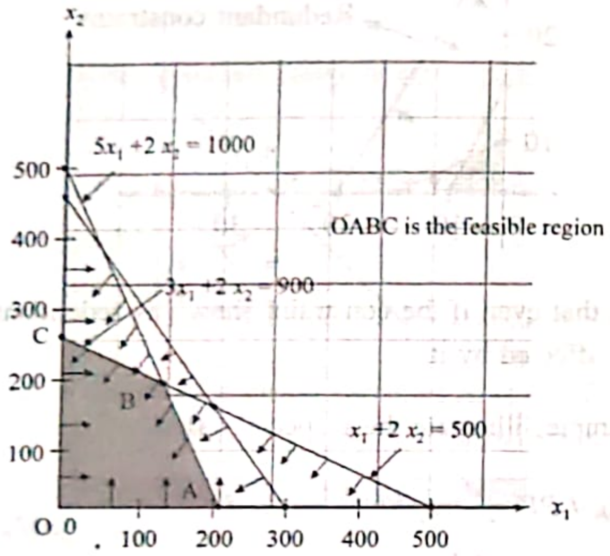
The maximum value of Z occurs at two vertices A and B,

That is $Z_A = Z_B = 20{,}000$

Hence, $x_1 = 200$, $x_2=0$

or

$x_1 = 125$, $x_2 = 187.5$ is the alternate optimal solution.



Scale: 1cm = 100 units

Note: *Whenever there exists more than one optimal solution the problem is said to have alternate optimal solution or infinite number of optimal solutions.*

32. *Solve $Z_{max} = 3x_1 + 2x_2$ by using graphical method*
Subject to $5x_1 + x_2 \geq 10$, $x_1 + x_2 \geq 6$, $x_1 + 4x_2 \geq 12$ and $x_1, x_2 \geq 0$

Solution:

$5x_1 + x_2 = 10$ passes through (0, 10); (2, 0)

$x_1 + x_2 = 6$ passes through (0, 6); (6, 0)

$x_1 + 4x_2 = 12$ passes through (0, 3); (12, 0)

Plotting these co–ordinates on the graph, the feasible region is open as per the plot shown. The maximum value of Z occurs at infinity. Hence, the solution is unbounded.

Observation: When the objective function is of maximization type with all the constraints ≥ 0 then the solution is unbounded as there is no constraints on the resources.

Note: *If the objective is to minimise in above problem, then the feasible region is OABC (open envelope).*

The value of objective function

$Z = 3x_1 + 2x_2$ is

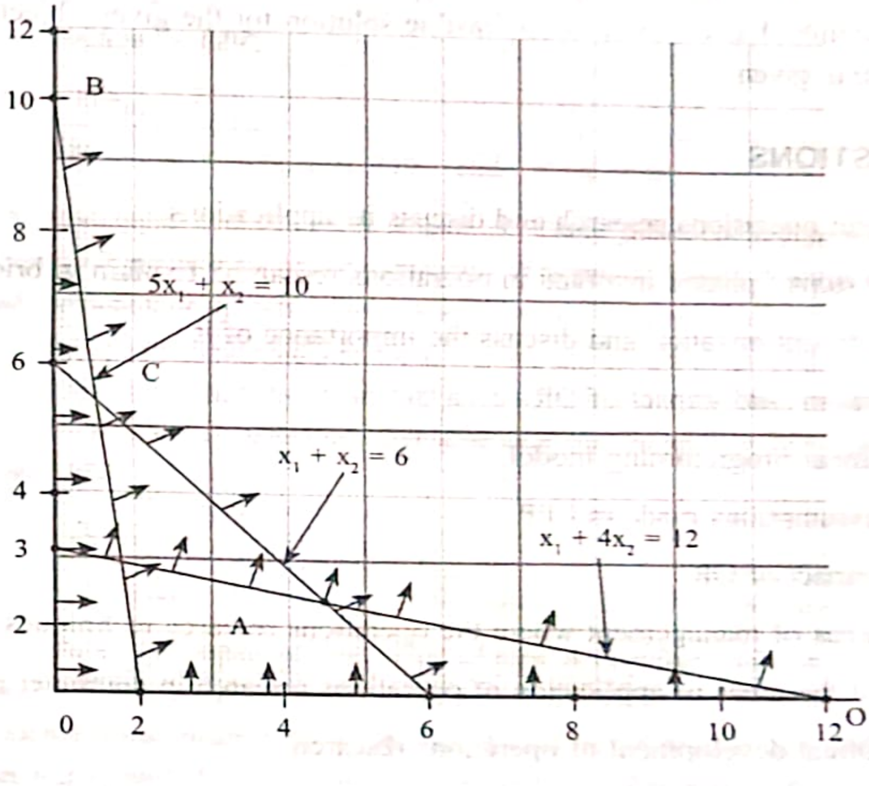
$Z_O = 3\ (12) + 2\ (0) = 36$

$Z_A = 3(4) + 2\ (2) = 16$

$Z_B = 3(0) + 2\ (10) = 20$

$Z_C = 3(1) + 2\ (5) = 13$

The minimum Z occurs at C whose value is 13. Hence, the values of x_1, x_2 are $x_1 = 1$, $x_2 = 5$.



Scale: 1cm = 2 units

33. ***Maximize*** $Z = 6x_1 + 4x_2$

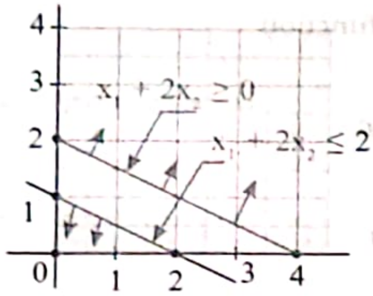
Subject to $x_1 + 2x_2 \le 2$

$x_1 + 2x_2 \ge 4$ ***and*** $x_1, x_2 \ge 0$

Solution:

$x_1 + 2x_2 = 2$ passes through (0, 1) ; (2, 0)

$x_1 + 2x_2 = 4$ passes through (0, 2) ; (4, 0)



Scale: 1cm = 1 unit

It is observed from the above plot that there is no common feasible region satisfying the given constraints. Hence, there is no feasible solution for the given objective function as per the constraints given.

Review Questions

1. Define the term operations research and discuss its applications.
2. What are the steps / phases involved in operations research? Explain in brief.
3. Define the term optimization and discuss the importance of it.
4. Explain the nature and impact of OR.
5. Explain the linear programming model.
6. List out the assumptions made in LPP.
7. Explain the impact of OR.
8. Discuss the areas of management where the operations research techniques are used.
9. Name some of the areas of application of operations research in computer engineering.
10. Give the historical development of operations research.
11. What are the advantages and limitations of operations research studies.
12. Give the characteristics / nature of operations research.
13. Explain mathematical formation of linear programming problems.
14. Explain linear programing model.
15. What is meant by LPP? Give the formulation of an LPP.
16. Write a note on the following with reference to LPP

    i) Decision variables ii) objective function iii) Constraints

17. Explain the steps involved in graphical method of solving a LPP.
18. Explain the following related to a linear programming problem

    i) Feasible solution ii) No solution iii) Unbounded solution iv) Feasible region

    v) Optimal solution

PROBLEMS

1. *A company produces two types of leather belts A and B. A is of superior quality, and B is of inferior quality. The respective profits are ₹20/– and ₹10/– per belt. The supply of raw materials is sufficient for making 1000 belts per day. For belt A, a special type of buckle is required and 500 are available per day. There are 750 buckles available for belt B per day. Formulate the problem as LPP.*

Ans: *Max.* $Z = 20x_1 + 10x_2$

S T $x_1 + x_2 \leq 1000$ $x_1 \leq 500, x_2 \leq 750$ and $x_1, x_2 \geq 0$

2. *A company manufactures two products A and B. These products are processed in the same machine. It takes 10 minutes to process one unit of product A and 2 minutes for each unit of product B and the machine operates for a maximum of 35 hours in a week. Product A requires 1 kg and B 0.5 kg of raw material per unit, the supply of which is 600 kg per week. Market constraints on product B is known to be 800 units every week. Product A costs ₹10/– per unit and sold at ₹20/– product B costs ₹6/– per unit and can be sold at ₹8/– per unit. Formulate the problem as LPP.*

Ans: *Max.* $Z = 10x_1 + 2x_2$

S T $10x_1 + 2x_2 \leq 35$ hours $x_1 + 0.5\,x_2 \leq 600$ kgs $x_2 \geq 800$ units and $x_1, x_2 \geq 0$

3. *A Company produces two types of hats. Hat of type A requires twice as much labor time as the second hat B. If the company produces only hat B then it can produce a total of 500 hats a day. The market limits daily sales of the hat A and hat B to 150 and 250 hats respectively. The profits on hat A and B is ₹8/– and 5/– respectively. Formulate the problem as LPP.*

Ans: *Min.* $Z = 8x_1 + 5x_2$

S T $2x_1 + x_2 \leq 500$ $x_1 \leq 150, x_2 \leq 250$ and $x_1, x_2 \geq 0$

4. *A computer company produces two models of computers, model J and K which fetch profit of ₹700 and ₹500 per unit respectively. Each unit of model J takes 4 hours of assembly time and two hours of testing time while each unit of model K requires 3 hours of assembly time and one hour of testing. In a given month, the total number of hours available for assembly is 210 hours and for inspection is 90 hours. Find how many units of each of the models to be produced in such a way that the total profit is maximum.*

Ans: ***Number of model 'J' computers = 30.***

Number of model 'K' computers = 30 and Z_{max} = 36,000.

5. ***A manufacturer produces two products, Klunk and Klick. Klunk has a contribution of ₹ 30/– per unit and Klick ₹ 40/– per unit. The manufacturer wishes to establish the weekly production plan which maximizes contribution. Production data are as follows:***

	PER – UNIT		
	Machining (Hours)	Labour (Hours)	Material (Kgs)
Klunk	4	4	1
Klick	2	6	1
Total availability per week	100	180	40

Because of a trade agreement, sales of Klunk is limited to a weekly maximum of 20 units and to honour an agreement with an old and established customer at least 10 units of Klick must be sold per week.

Formulate the L.P model, in the standard format.

Ans: $Z_{max} = 30\,x_1 + 40\,x_2$

Subject to the constraints

$4\,x_1 + 2\,x_2 \le 100$ $4\,x_1 + 6\,x_2 \le 180$ $x_1 + x_2 \ge 40$, $x_1 \le 20$, $x_2 \ge 10$ and $x_1, x_2 \ge 0$

6. ***Solve the following LPP graphically***

$Z_{max} = 3x_1 + 5x_2$

Subject to $x_1 + 2\,x_2 \le 2000$ $x_1 + x_2 \le 1500$ $x_2 \le 600$ and $x_1, x_2 \ge 0$

Ans: $Z_{max} = 5500$, $x_1 = 1000$ and $x_2 = 500$

7. ***Use graphical method to solve the following LPP***

$Z_{min} = 4x_1 + x_2$

Subject to the constraints

$3x_1 + x_2 = 3$ $4x_1 + 3x_2 \ge 6$ $x_1 + 2x_2 \le 3$ and $x_1, x_2 \ge 0$

Ans: $Z_{min} = 3$, $x_1 = 0$, $x_2 = 3$

8. $Z_{max} = 5x_1 + 8x_2$, *obtain* x_1, x_2

Subject to $3x_1 + 2x_2 \leq 36$

$x_1 + 2x_2 \leq 20 \quad 3x_1 + 4x_2 \leq 40$ and $x_1, x_2 \geq 0$

Ans: $x_1 = 2, x_2 = 9$ *and* $Z_{max} = 82$

9. $Z_{max} = 8x_1 + 16x_2$

Subject to $x_1 + x_2 \leq 200, \quad x_2 \leq 125, \quad 3x_1 + 6x_2 \leq 900$

Ans: $x_1 = 50, x_2 = 125$ *or* $x_1 = x_2 = 100$ *and* $Z_{max} = 240$

10. $Z_{min} = 3x_1 - 2x_2$, *solve for* x_1, x_2

Subject to $-2x_1 + 3x_2 \leq 9$

$x_1 - 5x_2 \geq -20$ and $x_1, x_2 \geq 0$

Ans: *Unbounded solution*

11. *Using graphical method solve the LPP*

Maximize $Z = 5x_1 + 4x_2$

Subject to $6x_1 + 4x_2 \leq 24$

$x_1 + 2x_2 \leq 6 \; -x_1 + x_2 \leq 1 \; x_2 \leq 2$ and $x_1, x_2 \geq 0$

Ans: $x_1 = 3, x_2 = 1.5$ *and* $Z_{max} = 21$

12. $Z_{max} = 3x_1 + 5x_2$, *solve for* x_1, x_2

Subject to $x_1 \leq 4$

$2x_2 \leq 12 \quad 3x_1 + 2x_2 \leq 18$ and $x_1, x_2 \geq 0$

Ans: $x_1 = 2$

$x_2 = 6$ and $Z_{max} = 36$

13. *Solve the following LPP by graphical method.*

$Z_{max} = 40x_1 + 35x_2$

Subject to,

$2x_1 + 3x_2 \leq 60$,

$4x_1 + 3x_2 \leq 96$ and $x_1, x_2 \geq 0$

Ans: $x_1 = 18, \quad x_2 = 8$ *and* $Z_{max} = 1000$

14. *Maximize* $Z = 6x_1 + 4x_2$

Subject to the constraints

$2x_1 + 4x_2 \leq 4$, $4x_1 + 8x_2 \geq 16$ and $x_1, x_2 \geq 0$

Ans: *No feasible solution*

15. *Use graphical method to solve the following LPP*

$Z_{min} = 1.5x_1 + 2.5x_2$

Subject to the constraints $x_1 + 3x_2 \geq 3$, $x_1 + x_2 \geq 2$ and $x_1, x_2 \geq 0$

Ans: $x_1 = 1.5$, $x_2 = 0.5$ and $Z_{min} = 3.5$

16. *Solve* $Z_{max} = x_1 + x_2$

Subject to $x_1 + x_2 \leq 1$, $-3x_1 + x_2 \geq 3$, $x_1, x_2 \geq 0$

Ans: Converting the inequalities into equations, we have

$x_1 + x_2 = 1$, $-3x_1 + x_2 = 3$ and they pass through (0,1), (1, 0) and (0, 3), (–1, 0) respectively. Plotting these on the graph, it can be observed that there is no common feasible region satisfying all the constraints. Hence the problem cannot be solved. In other words, the given LPP has **no solution** (infeasible solution).

17. *Solve* $Z_{min} = 3x_1 + 2x_2$ *by using graphical method*

Subject to $5x_1 + x_2 \geq 10$, $x_1 + x_2 \geq 6$

$x_1 + 4x_2 \geq 12$, $x_1, x_2 \geq 0$

Ans: $Z_{min} = 13$, $x_1 = 1$, $x_2 = 5$

18. *Solve* $Z_{max} = 500x_1 + 150$ ... *..bject to* $x_1 + x_2 \leq 60$

$2500x_1 + 50x_2 \leq 50,000$

Ans: $Z_{max} = 1000$

19. *Use graphical method to solve the problem*

Solve $Z_{max} = 2x_1 + x_2$ *by using graphical method*

Subject to $x_2 \leq 10$

$2x_1 + 5x_2 \leq 60$, $x_1 + x_2 \leq 18$, $3x_1 + x_2 \leq 44$ and $x_1, x_2 \geq 0$

Ans: $Z_{max} = 31$, $x_1 = 13$, $x_2 = 5$

20. *Use the graphical method to solve the following LPP.*

Minimise $Z = 1.5x_1 + 2.5x_2$ *subject to the constraints* $x_1 = 3x_2 \geq 3$, $x_1 + x_2 \geq 2$ *and* $x_1, x_2 \geq 0$

Ans: $x_1 = 1.5$, $x_2 = 0.5$, $Z_{min} = 3.5$