CENGAGE

4e

OPERATIONS RESEARCH

Designed for Computer Science Students

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2 Simplex Method - 1

3 Simplex method - 2, Duality Theory

4. Transportation and Assignment Problems

5 Game Theory and Metaheuristics

14. *Maximize* $Z = 6x_1 + 4x_2$

Subject to the constraints

$2x_1 + 4x_2 \le 4$, $4x_1 + 8x_2 \ge 16$ and $x_1, x_2 \ge 0$

Ans: *No feasible solution*

15. *Use graphical method to solve the following LPP*

$Z_{min} = 1.5x_1 + 2.5x_2$

Subject to the constraints $x_1 + 3x_2 \ge 3$, $x_1 + x_2 \ge 2$ and $x_1, x_2 \ge 0$

Ans: $x_1 = 1.5$, $x_2 = 0.5$ and $Z_{min} = 3.5$

16. *Solve* $Z_{max} = x_1 + x_2$

Subject to $x_1 + x_2 \le 1, -3x_1 + x_2 \ge 3$, $x_1, x_2 \ge 0$

Ans: Converting the inequalities into equations, we have

$x_1 + x_2 = 1, -3x_1 + x_2 = 3$ and they pass through (0,1), (1, 0) and (0, 3), (−1, 0) respectively. Plotting these on the graph, it can be observed that there is no common feasible region satisfying all the constraints. Hence the problem cannot be solved. In other words, the given LPP has **no solution** (infeasible solution).

17. *Solve* $Z_{min} = 3x_1 + 2x_2$ *by using graphical method*

Subject to $5x_1 + x_2 \ge 10$, $x_1 + x_2 \ge 6$

$x_1 + 4x_2 \ge 12$, $x_1, x_2 \ge 0$

Ans: $Z_{min} = 13$, $x_1 = 1$, $x_2 = 5$

18. *Solve* $Z_{max} = 500\,x_1 + 150$ [illegible]*bject to* $x_1 + x_2 \le 60$

$2500x_1 + 50x_2 \le 50{,}000$

Ans: $Z_{max} = 1000$

19. *Use graphical method to solve the problem*

Solve $Z_{max} = 2x_1 + x_2$ *by using graphical method*

Subject to $x_2 \le 10$

$2x_1 + 5x_2 \le 60$, $x_1 + x_2 \le 18$, $3x_1 + x_2 \le 44$ and $x_1, x_2 \ge 0$

Ans: $Z_{max} = 31$, $x_1 = 13$, $x_2 = 5$

20. *Use the graphical method to solve the following LPP.*

Minimise $Z = 1.5x_1 + 2.5x_2$ *subject to the constraints* $x_1 = 3x_2 \ge 3$, $x_1 + x_2 \ge 2$ *and* x_1, $x_2 \ge 0$

Ans: $x_1 = 1.5$, $x_2 = 0.5$, $Z_{min} = 3.5$

2

Simplex Method - 1

Learning objectives

After Studying this chapter, you should be able to

- understand the essence of simplex method
- set-up simplex tables and solve LP problems using simplex algorithm
- understand the tie breaking in simplex method
- distinguish the slack, surplus and artificial variables
- solve LP problems using Big-M method and two phase method

2.1 INTRODUCTION

Graphical method used to solve a Linear Programming Problem is limited to two decision variable problems. But most real life problems when formulated as LP model will have more than two decision variables. Thus, we need a more efficient method to suggest an optimal solution of such problems. A more general method known as "Simplex Method" is suitable for solving Linear Programming Problems with a larger number of variables. It is an iterative process, progressively approaching and ultimately reaching the maximum or minimum values of the objective function. The method was developed by G. B. Dantizig in 1947, an American Mathematician.

2.2 THE ESSENCE OF SIMPLEX METHOD

Simplex method is an algebraic procedure. However, its underlying concepts are geometric.

Understanding these geometric concepts provides an intuitive insight of how the simplex method operates and what makes it so efficient.

The geometric concepts are related to the algebra of the simplex method. In graphical method of solving an LPP, we used to identify a common region known as feasible region satisfying all the constraints. The optimal solution used to occur at some vertex of the Fesible region.

If the optimal solution was not unique, the optimal points were on an edge. Essentially the problem is that of finding the particular vertex for the convex region which corresponds to the optimal solution.

The simplex method provides an algorithm which consists in moving from one vertex of the region of feasible solutions to another vertex in such a way that the value of the objective function at the succeeding vertex is improved than at the previous vertex.

This procedure of iteration from one vertex to another is repeated till the optimal solution is obtained. Thus, the geometric concepts are related to the algebra on which the simplex method is works.

Application of Simplex Method

Simplex method can be used to solve the problems having two or more variables, where as the graphical method is confined to the problems having only two variables.

Graphical method is a geometric approach, where as Simplex method is algebraic approach. The simplex method can be computerised easily by programming it.

2.3 BASIC TERMS / DEFINITIONS

i. **Slack variable**

A variable added to the left hand side of a constraint (less than or equal to) to convert the constraint into an equation is called a Slack Variable.

Example

If the constraint given is $2x_1 + 3x_2 \leq 8$, then

$2x_1 + 3x_2 + u_1 = 8$ is the equality (equation)

Where u_1 is a slack variable.

ii. **Surplus variable**

A variable subtracted from the left hand side of the constraint (greater than or equal to) and to convert it into an equality is called a surplus variable.

Example

If the constraint given is, $3x_1 + 4x_2 \geq 10$ then

$3x_1 + 4x_2 - s_1 = 10$ is the equation; where, s_1 is the surplus variable.

iii. **Basic Solution**

The initial solution obtain after setting the basic variables as zeros is basic solution

It is the unique solution resulting from setting (n – m) variables equal to zero.

Where, m = number of simultaneous linear equations

n = number of variables.

iv. **Basic Feasible Solution**

A basic solution which satisfies $x_i \geq 0, i = 1, 2, \ldots\ldots n$ *is called as a basic feasible solution (which indicates that the decision variables can not be negative.*

v. **Optimal Solution**

Any basic feasible solution which optimizes (minimizes or maximizes) the objective function of a general LP problem is known as an optimal solution.

2.4 STANDARD FORM OF AN LP PROBLEM (CHARACTERISTICS OF LPP)

Simplex method to solve LP problem requires the problem be converted into standard form. The standard form of the LP problem should have the following characteristics.

i. *All the constraints should be expressed as equations by adding slack or surplus variables.*

ii. *The right hand side of each constraint should be made non negative; if it is not, this should be done by multiplying both sides of the resulting constraint by –1.*

iii. *The objective function should be of the maximization type (if it is not, should be converted by multiplying with –1).*

Worked Examples

1. *Obtain all the basic solutions to the following system of linear equations. Is the non - degenerate solution feasible?*

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

Which of them are basic feasible solutions.

Solution:

We have, number of unknowns (variables) = 4 and number of equations = 2. There will be $4C_2$ (=6) different possible basic solutions

Whenever all the values of basic variables ≥ 0, then the solution is feasible.

S. No.	Basic variable	Non - basic variable	Value of basic variables	Is the solution feasible?
1	x_1, x_2	$x_3 = x_4 = 0$	$2x_1 + 6x_2 = 3$ $6x_1 + 4x_2 = 2$ $x_1 = 0, \quad x_2 = \frac{1}{2}$	Yes
2	x_1, x_3	$x_2 = x_4 = 0$	$2x_1 + 2x_3 = 3$ $6x_1 + 4x_3 = 2$ $x_1 = -2, \quad x_3 = \frac{7}{2}$	No
3	x_1, x_4	$x_2 = x_3 = 0$	$2x_1 + x_4 = 3$ $6x_1 + 6x_4 = 2$ $x_1 = \frac{8}{4}, \quad x_4 = -\frac{7}{3}$	No
4	x_2, x_3	$x_1 = x_4 = 0$	$6x_2 + 2x_3 = 3$ $4x_2 + 4x_3 = 2$ $x_2 = \frac{1}{2}, \quad x_3 = 0$	Yes
5	x_2, x_4	$x_1 = x_3 = 0$	$6x_2 + x_4 = 3$ $4x_2 + 6x_4 = 4$ $x_1 = \frac{1}{2}, \quad x_4 = 0$	Yes
6	x_3, x_4	$x_1 = x_2 = 0$	$2x_1 + x_4 = 3$ $4x_3 + 6x_4 = 2$ $x_3 = 2, x_4 = -1$	No

2. ***Find all the basic solutions of the following system of equations identifying in each case the basic and non basic variables.***

$2x_1 + x_2 + 4x_3 = 11$

$3x_1 + x_2 + 5x_3 = 14$

Solution:

Number of equations = 2

Number of variables = 3

$\therefore$ There are $3C_2$ possible ways of getting different basic solutions $3C_2 = \frac{3 \times 2}{2} = 3$ ways

When, $x_3 = 0$ $\quad$ $2x_1 + x_2 = 11,$ $\quad$ $3x_1 + x_2 = 14$

On solving $\quad$ $x_1 = 3, x_2 = 5$

$x_1 = 0$ $\quad x_2 + 4x_3 = 11,$ $\quad x_2 + 5x_3 = 14$

On solving $\quad x_2 = -1, x_3 = 3$

$x_2 = 0$ $\quad 2x_1 + 4x_3 = 11,$ $\quad 3x_1 + 5x_3 = 14$

On solving $\quad x_1 = \frac{1}{2}, \quad x_3 = \frac{5}{2}$

S. No	Basic variable	Non – basic variable	Value of basic variables	Is the solution feasible?
1	x_1, x_2	x_3	$x_1 = 3, x_2 = 5$	Yes
2	x_2, x_3	x_1	$x_2 = 3, x_3 = -1$	No
3	x_1, x_3	x_2	$x_1 = \frac{1}{2}, \quad x_3 = \frac{5}{2}$	Yes

Infeasible solution

Whenever the design variable or basic variable assumes a negative value the solution is stated as infeasible.

2.5 THE SETTING UP AND ALGEBRA OF SIMPLEX METHOD

Computational procedure of the method requires the construction of the 'simplex tableau'. The initial simplex table is formed by writing out the coefficients and constraints of a Linear Programming Problem in a systematic tabular form.

Step 1

Check whether the given objective function is to be maximized or minimized.

If the objective function is to be maximized, take it in the given form itself. If it is to be minimized, then convert into maximization from.

The objective function to be maximized will be of the form.

$Z = c_1x_1 + c_2x_2 + \ldots c_kx_k$

Step 2

Express the problem in the standard form by introducing appropriate slack / surplus variables. In standard from, the constraints would consist of *m* equations in *n* variables (including slack variables). The equations will be of the following form

$$a_{11}x_1 + a_{12}x_2 + \ldots.. + u_1 + o.u_2 + \ldots.. + o.u_p = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots.. + 0.u_1 + 1.u_2 + \ldots.. + o.u_p = b_2 \quad (2)$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots + o\,u_1 + o.u_2 + \ldots\ldots + o.u_p = b_m$$

Here, u_1, u_2, ...u_p are slack variables

Step 3

Equating the basic variables to zero obtain an initial basic feasible solution

Step 4

Simplex method in tabular form

Basis	C_B	x_1	x_2	$x_3 \ldots$	u_1	u_2	$u_3 \ldots$	B
u_1	c_1	a_{11}	a_{12}	$a_{13} \ldots$	1	0	0	B_1
u_2	c_2	a_{21}	a_{22}	$a_{23} \ldots$	0	1	0	B_2
u_3	c_3	a_{31}	a_{32}	$a_{33} \ldots$	0	0	1	B_3
...	...	...	...	...	...	...	...	...
C_j		C_1	C_2	$C_3\ldots$	0	0	0...	–
Z_j		Z_1	Z_2	Z_3	C_1	C_2	$C_3\ldots$	$\sum C_k B_k$
$C_j - Z_j$		$C_1 - Z_1$	$C_2 - Z_2$	$C_3 - Z_3$				

In this table

i. *The basic variables are listed in the basis column.*

ii. *The coefficients of the variables u_1, u_2, u_3 in the objective function are listed in column C_B.*

iii. *The coefficients of x_1, x_2, x_3, ..., u_1, u_2, u_3, ... in equation (2) are listed in columns x_1, x_2, x_3, ..., u_1, u_2, u_3, ... respectively. The coefficients of x_1, x_2, x_3, ..., form what is called the body matrix, and the coefficients of u_1, u_2, u_3, ... constitute a unit matrix.*

iv. *The values B_1, B_2, B_3,....... of the basic variables x_1, x_2, x_3, ..., are listed in column B.*

v. *The coefficients of x_1, x_2, x_3, ..., u_1, u_2, u_3, ... in Z are listed in row C_j.*

vi. *The entries Z_1, Z_2, Z_3, ..., in the row S are computed by the formulae.*

$$Z_1 = a_{11}\,C_1 + a_{21}\,C_2 + a_{31}\,C_3 + \ldots = \sum_k a_{1k}C_k$$

$$Z_2 = a_{21}\,C_1 + a_{22}\,C_2 + a_{32}\,C_3 + \ldots = \sum_k a_{2k}C_k \text{ , and so on}$$

vii. *The entries in $(C_j - Z_j)$ row are determined as indicated in the table.*

This row is called the net cell evaluation row.

Step 5

Examine the entries in $(C_j - Z_j)$ row, namely: $C_1 - Z_1$, $C_2 - Z_2$, $C_3 - Z_3$, ... If all of these are ≤ then the basic solution x_1, x_2, x_3, ..., is an optimal solution, and the problem is solved. If any $C_j - Z_j$ value is not ≤ 0, then proceed to next step.

Step 6

Among the positive entries in $(C_j - Z_j)$ row, identify the one which is maximum. (If only one entry is positive, identify that entry. If more than one entry is positive select the highest one.) The column containing this entry is called the pivotal column (PC) or the key column.

Now, divide the entries in column B by the corresponding entries in the pivotal column (leaving out those which are zero).

Among the ratios so obtained, identify the one which is non – negative and minimum (least). The row containing this ratio is called the pivotal row (PR) or the key row. The entry common to the pivotal column and the pivotal row is called the pivotal element (PE) or the key element.

The basic variable corresponding to the pivotal row is called the leaving variable (or departing variable) and the variable corresponding to the pivotal column is called the entering variable (or the arriving variable). This means that, for the next stage of work, the basic solution is obtained by replacing the departing variable with the arriving variable.

Step 7

After the departing and arriving variables are identified as explained above, divide every entry in the pivotal row by the pivotal element to make the pivotal element as 1. There after, make all the remaining entries in the pivotal column zero by elementary row operations and obtain modified simplex table.

Step 8

After constructing the modified table as explained above, write down the C_j row, Z_j row and $(C_j - Z_j)$ row, (There would be no change in C_j row). The resulting table is called the second simplex table. Analyse this table as explained in step 5 and repeat the process until an optimal (or an unbounded) solution is obtained.

2.5.1 Steps of Simplex Method in Brief

i. *Formulate the objective function and constraints*

ii. *If the objective is to minimize, convert into standard form that is, maximization (by multiplying the coefficients of objective function with –ve sign)*

iii. *Add slack variables to convert each constraint to an equation*

iv. *Setup the first starting solution*

v. *Check solution for optimality. If optimal, stop otherwise continue.*

vi. *Select a variable to enter to improve the solution*

vii. *Select a variable to leave the basis*

viii. *Perform row operations to complete the solution*

ix. *Return to step 4 and continue the procedure until optimality is obtained.*

Outline of Simplex Method

S1: *Determine the starting basic feasible solution.*

S2: *Select an entering variable using row operations. If there is no entering variable, the solution is optimaland the Stop process. Else go to Step 3.*

S3: *Select a leaving variable using therowopeartions based on ratio.*

S4: *Detaermine the new basic solution using the row operations. Go to Step 2.*

2.5.2 Steps in Performing row Operations

i. *Identify the PE, which is the intersection of the PR and PC.*

ii. *Divide the PR, element by element, by the PE. Enter this new row in the next tableau in the same row position.*

iii. *Reduce all other elements in the PC in the next tableau to zero by multiplying the new row formed in step 2 by the negative of the row's elements in the PC of the present tableau and adding this transitional row to the row being modified.*

iv. *New pivot equations =* $\frac{\text{old equations element}}{\text{pivot element}}$ *and the other equations above or below to the pivot element equation = old element – [its entering coefficient of the column × corresponding new pivot equation]*

Let us understand the simplex method through the following examples.

3. *Solve the following LP problem by the Simplex method*

Maximize $Z = x_1 + 3x_2$

Subjected to $x_1 + 2x_2 \le 10$

$0 \le x_1 \le 5$ *and* $0 \le x_2 \le 4$

Solution:

Step 1

The objective function is in the standard form (maximization) and hence there is no need for modification.

The design (decision) variables are x_1, x_2

Step 2

Converting the inequalities as equations we get,

$x_1 + 2x_2 + u_1 = 10$

$x_1 + u_2 = 5$

$x_2 + u_3 = 4$

$x_1 \geq 0,\ x_2 \geq 0$

Where $u_1,\ u_2,\ u_3$ are the slack variables.

Step 3

Set the design variables x_1, x_2 to zero, so that the starting basic solution is

$u_1 = 10, \qquad u_2 = 5, \qquad u_3 = 4$

Step 4

In terms of all these variables the objective function is,

$$Z_{max} = x_1 + 3x_2 + 0u_1 + 0u_2 + 0u_3 \qquad (1)$$

(Add slack variables with '0' as co-efficient)

Step 5

Prepare the starting simplex table as shown below,

↓ E.V

Basis	C_B	x_1	x_2	u_1	u_2	u_3	B	**Ratio** (θ) **=** B **/ Elements of** PC
u_1	0	1	2	1	0	0	10	10 / 2 = 5
u_2	0	1	0	0	1	0	5	5 / 0 (not defined)
L.V ← u_3	0	0	$\boxed{1}$ PE	0	0	1	4	4 /1 = 4 ← PR
C_j		1	3	0	0	0		
Z_j		0	0	0	0	0	0	
$C_j - Z_j$		1	3	0	0	0	–	

↑

PC

In the above table,

i. *Column 'Basis' contains the basic variables. (In the first simplex table they are slack variables that is,* u_1, u_2, u_3*).*

ii. *Column* C_B *contains the co efficient of the variables (*u_1, u_2, u_3*) in the objective function.*

iii. *Column* x_1, x_2, x_3 *respectively contain the co efficients of* x_1, x_2, x_3 *in the constraint equations.*

iv. *Columns* u_1, u_2, u_3 *respectively contain the co–efficient of* u_1, u_2, u_3 *in the constrains equations. These coefficients form a unit matrix.*

v. *Column 'B' contains the values of R.H.S. of the constraints.*

vi. *Row '*C_j*' contains the co–efficients of all variables in the objective function represented as 1.*

Row 'Z_j' contains the entries determined as explained below

For each column, multiply each entry in that column by the corresponding entry in the column C_B and add up the values.

For example the entry in the column x_1 is determined as $2 \times 0 + 0 \times 0 + 3 \times 0 = 0$.

Step 6

If all $C_j - Z_j$ values ≤ 0 then there is no need to proceed the next table (the solution is optimum).

Here it is not, so we proceed to the next step.

Step 7

Among the positive entries in $(C_j - Z_j)$ row, identify the one which is maximum. The column containing this entry is called the pivotal column (PC).

Now, divide the entries in column B by the corresponding entries in the pivotal column (leaving out those which are zero). Among the ratios θ so obtained, identify the one that is positive and minimum. The row containing this ratio is called the pivotal row (PR). The intersection of the pivotal column and the pivotal row is called the pivotal element (PE). The basic variable corresponding to the pivotal row is called the leaving variable (LV) and the variable corresponding to the pivotal column is called as entry variable (EV), i.e. in the next table LV will be replaced with EV.

Note:

i. *If all the ratios (θ) are non positive, we conclude that the problem has* ***unbounded solution*** *and no further working process is required.*

ii. *Consider the elements of pivot column having only with positive sign.*

Step 8

Divide every entry in the pivotal row by the pivotal element. (To make the pivotal element as unity). Thereafter make all the remaining entries in the pivotal column zero by row operations that is by the technique used in Gauss elimination or Gauss-Jordan methods.

Note:

Modified simplex table is based on the steps 7, 8

If the pivot element is not unity, divide all the elements of pivotal row by pivot element.

Step 9

The resulting table is second simplex table. Analyze the table as explained in step 6 and repeat the procedure until an optimal (or an unbounded) solution is obtained.

In the first simplex table we observe that the x_2 column is PC, u_1 row is PR and 1 is PE. Therefore, u_3 is leaving variable and x_2 is the arriving (entry) variable, so the basis for the next simplex table is (u_1, u_2, x_2).

Second Simplex Table

To obtain next simplex table for the given problem the following row operations are used.

R_3 (new) = R_3 (old) / pivot element, R_2 (new) = R_2 (old), as R_2 (old) already contains zero below the pivot element.

R_1 (new) = R_1 (old) – 2 × R_3 (new) where 2 is the coefficient of PC that should be converted as zero.

↓ EV

Basis	C_B	x_1	x_2	u_1	u_2	u_3	B	Ratio (θ)	
LV ← u_1	0	$\boxed{1}$ PE	0	1	0	– 2	2	2 / 1 = 2	← PR
u_2	0	1	0	0	1	0	5	5 / 1 = 5	
x_2	3	0	1	0	0	1	4	4 / 0 (not defined)	$-\infty$
C_j		1	3	0	0	0	–	–	
Z_j		0	3	0	0	3	12	–	
$C_j - Z_j$		1	0	0	0	– 3	–	–	

↑
PC

Again, as all $C_j - Z_j$ are not ≤ 0, the solution is not yet optimum.

Third Simplex Table R_2 (new) = R_2 (old) – 1 × R_1 (new)

Basis	C_B	x_1	x_2	u_1	u_2	u_3	B
x_1	1	$\boxed{1}$ PE	0	1	0	– 2	2 ← PR
u_2	0	0	0	– 1	0	2	3
x_2	3	0	1	0	0	1	4
C_j		1	3	0	0	0	–
Z_j		1	3	1	0	1	14
$C_j - Z_j$		0	0	– 1	0	– 1	–

We observe that in the above table, none of the entries in ($C_j - Z_j$) row are in positive. We therefore stop the process (optimality is obtained).

We find $x_1 = 2$, $x_2 = 4$ (entries in column B corresponding to x_1, x_2)

Hence, the optimum solution (that is, Z_{max}) is

$Z_{max} = x_1 + 3\ x_2 = 2 + 3\ (4) = 14$

Note

$Z_j - C_j$ may also be considered instead $Cj - Zj$, to conclude the optimality considering highest negative value. In this case, all $Z_j - C_j$ values should be ≥ 0.

Observation while moving from one table to another table

i. ***Pivot element is unity, if not divide the entire row by the pivot element (the division is from x_1 to u_n).***

ii. ***Other elements of pivotal column are converted as zeros by row operations.***

iii. ***Leaving variable is replaced with entry variable with its coefficient.***

iv. ***Row operations need not be unique, the operation should nullify the elements of basic PC keeping PE as unity (1). Hence, the equations for row operations are not shown for all the problems.***

4. *Use simplex method to solve the LPP*
Max $Z = 3x_1 + 2x_2$
Subject to $x_1 + x_2 \leq 4$, $x_1 - x_2 \leq 2$ *and* $x_1, x_2 \geq 0$

Solution:

By introducing the slack variables u_1, u_2 we get,

$x_1 + x_2 + u_1 = 4$

$x_1 - x_2 + u_2 = 2$

$Z_{max} = 3x_1 + 2x_2 + 0u_1 + 0u_2$ as the standard LPP

Where $x_1, x_2, u_1, u_2 \geq 0$

First Simplex Table

↓ EV

Basis	C_B	x_1	x_2	u_1	u_2	B	**Ratio**
u_1	0	1	1	1	0	4	4 / 1 = 4
LV ← u_2	0	$\boxed{1}$ PE	− 1	0	1	2	2 / 1 = 2 ← PR
C_j		3	2	0	0	–	–
Z_j		0	0	0	0	0	–
$C_j - Z_j$		3	2	0	0	–	–

↑

PC

Second Simplex Table

↓ EV

Basis	C_B	x_1	x_2	u_1	u_2	B	Ratio
LV ← u_1	0	0	$\boxed{2}$ PE	1	– 1	2	2 / 2 = 1 ← PR
x_1	3	1	– 1	0	1	2	2 / – 1 = –2(not to be considered)
C_j		3	2	0	0	–	–
Z_j		3	– 3	0	3	6	–
$C_j – Z_j$		0	5	0	– 3	–	–

↑

PC

As all $C_j - Z_j$ are not lesser than or equal to zero the solution is not yet optimum. Hence, move to the next table revising the basis as $\{x_2\ x_1\}$ keeping the PE as unity, making other elements in PC zeros.

Third Simplex Table

Basis	C_B	x_1	x_2	u_1	u_2	B
x_2	2	0	$\boxed{1}$ PE	1 / 2	– 1 / 2	1
x_1	3	1	0	1 / 2	1 / 2	3
C_j		3	2	0	0	–
Z_j		3	2	5 / 2	1 / 2	11
$C_j – Z_j$		0	0	– 5 / 2	– 1 / 2	11

Since all $C_j - Z_j \leq 0$, the solution is optimum.

The optimal solution is max Z ~~= [illegible]~~

$x_1 = 1$ $x_2 = 3$
$Z_{max} = 11$

Note:

i. ***The simplex method discussed is applicable to maximisation problems only. If objective of the problem is minimisation then it is to be converted into maximisation form (standard form).***

ii. ***Row operations need not be unique, only thing that is important is pivot element should be unity and other elements of pivot column to be made as zeros.***

5. *Solve the following LPP by simplex method.*

$Z_{\min} = x_1 - 3x_2 + 2x_3$

Subject to $3x_1 - x_2 + 2x_3 \leq 7$, $-2x_1 + 4x_2 \leq 12$ *and* $-4x_1 + 3x_2 + 8x_3 \leq 10$

Solution:

As the given objective function is to minimize, it should be converted into maximization form.

$M = Z_{max} = -\left(Z_{min}\right)$

$M = -x_1 + 3x_2 - 2x_3$

$3x_1 - x_2 + 2x_3 + u_1 = 7$

$-2x_1 + 4x_2 + 0x_3 + u_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + u_3 = 10$

The modified objective function is

$Z*_{max} = -x_1 + 3x_2 - 2x_3 + 0u_1 + 0u_2 + 0u_3$

Setting the basic variables to zero, we get $u_1 = 7$, $u_2 = 12$ and $u_3 = 10$

First Simplex Table

↓ EV

Basis	C_B	x_1	x_2	x_3	u_1	u_2	u_3	B	Ratio
u_1	0	3	– 1	2	1	0	0	7	– 7
LV ← u_2	0	– 2	$\boxed{4}$ PE	0	0	1	0	12	3 ← PR
u_3	0	– 4	3	8	0	0	1	10	10 / 3
C_j		– 1	3	–2	0	0	0	–	
Z_j		0	0	0	0	0	0	0	
$C_j - Z_j$		– 1	3	– 2	0	0	0	–	

↑

PC

As all $C_j - Z_j$ are not lesser than or equal to zero the solution is not yet optimum. Hence, move to the next table revising the basis keeping the PE as unity, making other elements in PC zeros.

Second Simplex Table

↓ EV

Basis	C_B	x_1	x_2	x_3	u_1	u_2	u_3	B	Ratio
LV ← u_1	0	$\boxed{5/2}$ PE	0	2	1	1 / 4	0	10	4 ← PR
x_2	3	– 1 / 2	1	0	0	1 / 4	0	3	– 6
u_3	0	– 5 / 2	0	8	0	– 3 / 4	1	1	– 2/5
C_j		– 1	3	– 2	0	0	0		
Z_j		– 3 / 2	3	0	0	3 / 4	0	9	
$C_j - Z_j$		1 / 2	0	– 2	0	– 3 / 4	0		

↑

PC

As all $C_j - Z_j$ are not lesser than or equal to zero the solution is not yet optimum. Hence, move to the next table revising the basis keeping the PE as unity, making other elements in PC zeros.

Third Simplex Table

Basis	C_B	x_1	x_2	x_3	u_1	u_2	u_3	B
x_1	– 1	1	0	4 / 5	2/5	2 / 20	0	4
x_2	3	0	1	2 / 5	1/5	3 / 10	0	5
u_3	0	0	0	10	1	– 1 / 2	1	11
C_j		– 1	3	– 2	0	0	0	–
Z_j		– 1	3	2 / 5	1/5	4 / 5	0	11
$C_j - Z_j$		0	0	– 12 / 5	– 1/5	– 4 / 5	0	–

The solution is optimal since all $C_j - Z_j$ values are less than or equal to 0

Hence $x_1 = 4$, $x_2 = 5$, $x_3 = 0$

$M = Z_{max} = -4+15 = 11$

$M = -(Z_{min}) = Z_{max}$

$Z_{min} = -11$

6. *Solve the following LPP by the simplex method*

Minimize $Z = 2x_1 + 3x_2 + x_3$

Subject to $3x_1 + 2x_2 + x_3 \leq 3$

$2x_1 + x_2 + x_3 \leq 2$

$x_1, x_2, x_3 \geq 0$

Solution:

The constraints can be written as,

$3x_1 + 2x_2 + x_3 + u_1 = 3$

$2x_1 + x_2 + x_3 + u_2 = 2$

Convert the objective function into maximization form

$Z_{max} = -2x_1 - 3x_2 - x_3$

$Z_{modified} = -2x_1 - 3x_2 - x_3 + 0u_1 + 0u_2$

First Simplex Table

Basis	C_B	x_1	x_2	x_3	u_1	u_2	B
u_1	0	3	2	1	1	0	3
u_2	0	2	1	1	0	1	2
C_j		– 2	– 3	– 1	0	0	–
Z_j		0	0	0	0	0	0
$C_j - Z_j$		– 2	– 3	– 1	0	0	

As all the values of $C_j - Z_j \leq 0$

The solution is optimum. Hence, $x_1 = x_2 = x_3 = 0$

$Z_{min} = 0$, that is the minimum possible value is zero subject to the given constrains.

7. $Z_{max} = 6x_1 + 8x_2$

Subject to $2x_1 + 8x_2 \leq 16$

$2x_1 + 4x_2 \leq 8$

Solution:

The constraints can be written as,

$x_1 + 4x_2 \leq 8$

$x_1 + 2x_2 \leq 4$ (dividing by 2 on both sides, to keep in the possible simplified form)

Adding the slack variables u_1, u_2 we get :

$x_1 + 4x_2 + u_1 = 8$

$x_1 + 2x_2 + u_2 = 4$

The modified objective function is,

$Z^*_{max} = 6x_1 + 8x_2 + 0u_1 + 0u_2$

Simple Table-1

↓ EV

	Basis	C_B	x_1	x_2	u_1	u_2	B	Ratio	
	u_1	0	1	4	1	0	8	8/4 = 2	
← LV	u_2	0	1	$\boxed{2}$ PE	0	1	4	4/2 = 2	← PR
	C_j		6	8	0	0	–		
	Z_j		0	0	0	0	0		
	$C_j - Z_j$		6	8	0	0			

↑PC

The solution is not optimum as all Cj – Zj are not ≤ 0

The highest positive difference of Cj – Zj is 8

After obtaining the ratio of B/PC, there is a tie

To resolve the tie find,

$$\left[\frac{\text{The elements of unit matrix I column}}{\text{PC elemets}}\right]$$

$$\text{i.e., }\left[\frac{1}{4},\frac{0}{2}\right]=\left[\frac{1}{4},0\right]$$

The minimum value is 0. Hence, select u_2 as the LV rather than considering u_1 [Making PE as 1, other elements of PC as 0s by $R_1 - 2R_2$]

Simplex Table-2

↓ EV

	Basis	C_B	x_1	x_2	u_1	u_2	B	Ratio	
	u_1	0	–1	0	1	–2	0	–(Not considered as PC Element is –ve.	
LV ←	x_2	8	$\boxed{1/2}$ PE	1	0	1/2	2	← 4 PR	old Row/2
	C_j		6	8	0	0	–		
	Z_j		4	8	0	4			
	$C_j - Z_j$		2	0	0	–4			

↑ PC

The solution is not yet optimum as all $C_j - Z_j$ are not ≤ 0
Hence, moving to next iteration with usual procedure.

Simplex Table-3,

Basis	C_B	x_1	x_2	u_1	u_2	B	
u_1	0	0	2	1	–1	4	→ $R_1 + 2R_2$
x_1	6	1	2	0	1	4	→ old R*2
C_j		6	8	0	0	24	
Z_j		6	12	0	6	–	
$C_j - Z_j$		0	–4	0	–6		

The solution is optimum as all $C_j - Z_j \le 0$

The values of x_1, x_2 are $x_1 = 4_1$, $x_2 = 0$ which gives $Z_{max} = 6(4) + 8(0) = 24$

8. *Using simplex method of tabular form solve the LPP*

Maximize $z = 4x_1 + 3x_2 + 6x_3$

Subject to $2x_1 + 3x_2 + 2x_3 \leq 440$, $4x_1 + 3x_3 \leq 470$, $2x_1 + 5x_2 \leq 430$ *and* $x_1, x_2, x_3 \geq 0$

Solution:

Converting the in equalities as equations by adding slack variables

$2x_1 + 3x_2 + 2x_3 + u_1 = 440$

$4x_1 + 0x_2 + 3x_3 + u_2 = 470$

$2x_1 + 5x_2 + 0x_3 + u_3 = 430$

(where u_1, u_2, u_3 are the slack variables)

The modified objective function is $Z^*_{max} = 4x_1 + 3x_2 + 6x_3 + 0u_1 + 0u_2 + 0u_3$

Simplex Table-1

↓ EV

	Basis	C_B	x_1	x_2	x_3	u_1	u_2	u_3	*B*	*Ratio*	
	u_1	0	2	3	2	1	0	0	440	440/2 = 220	
← LV	u_2	0	4	0	$\boxed{3}$ PE	0	1	0	470	470/3 = 156.6	← PR
	u_3	0	2	5	0	0	0	1	430	–	
	C_j		4	3	6	0	0	0	–		
	Z_j		0	0	0	0	0	0	0		
	$C_j - Z_j$		4	3	6	0	0	0			

↑PC

The more highest positive value of $C_j - Z_j$ is 6 Hence, the column corresponding to this becomes PC.

Replace the LV with EV, keeping the PE as '1' we get the new table as shown below (simplex table – 2)

Note:

i. *In simplex table-2 the PE must be unity (1), other elements of the PC must be zeros. Now the 'PE' is made as '1' and this can be treated as new* R_2. *The other elements of PC (above/ below of the PE) must be nullified.*

ii. *Coincidently the element below PE is '0' hence, no need to operate the 3rd row. Thus third row remains as it is. But, the element above PE is having a value of '2'. To make it as zero, the following row operators may be used.*

$3R_1 - 2R_2$ (Both old) or

Old R_1 − 2 New R_2 (old row can be operated with new row having PE as '1']

Let us prefer the second choice which is relatively simple to operate

Simplex Table-2

↓ EV

	Basis	C_B	x_1	x_2	x_3	u_1	u_2	u_3	*B*	***Ratio***	
	u_1	0	−2/3	$\boxed{3}$ PE	0	0	−2/3	0	380/3	380/9 ← PR	[old R_1−2 New R_2]
(New R_2)	x_3	6	4/3	0	1	0	1/3	0	470/3	–	← [oldR_2/ R_3]
	u_3	0	2	5	0	0	0	1	430	430/5	
	C_j		4	3	6	0	0	0	–	–	
	Z_j		8	0	6	0	2	0	940	–	
	$C_j - Z_j$		−4	3	0	0	−2	0	–	–	

↑PC

In the simplex table – 2 one $C_j - Z_j$ is at positive level. Hence, the solution is not yet optimum. Hence, the column corresponding to this becomes PC with x_2 as EV. The ratio is least for the first row hence, the first row is PR, u_1 is the replacing variable.

We get the modified simplex table as given below

	Basis	C_B	x_1	x_2	x_3	u_1	u_2	u_3	*B*	
(Newt R_1)	x_2	3	−2/9	1*	0	0	−2/9	0	380/9	old R_1/3
	x_3	6	4/3	0	1	0	1/3	0	470/3	
	u_3	0	28/9	0	0	0	10/9	1	3680/9	old R_3 − 5 New R_1
	C_j		4	3	6	0	0	0	–	
	Z_j		22/3	3	6	0	4/3	0	3200/3	
	$C_j - Z_j$		−10/3	0	0	0	−4/3	0	–	

The solution is optimum as all $C_j - Z_j \leq 0$

The optimum values of the decision variables are,

$x_1 = 0$ (as it has not appeared in the basis)

$$x_2 = \frac{380}{9}, \quad x_3 = \frac{470}{3}$$

which gives

$$Z_{\max} = 4x_1 + 3x_2 + 6x_3$$

$$= 4(0) + 1\left(\frac{380}{9}\right) + 6\left(\frac{470}{3}\right)$$

$$= \frac{3200}{3}$$

9. ***Solve the following LPP by Simplex Method***

Max $Z = 3x_1 + 2x_2$

Subject to $4x_1 + 3x_2 \leq 12,$

$4x_1 + x_2 \leq 8,$

$4x_1 - x_2 \leq 8,$

$x_1, x_2 \geq 0$

Solution:

Convert the inequality of the constraints into equations by adding slack variables u_1, u_2, u_3, then

Max. $Z = 3x_1 + 2x_2 + 0u_1 + 0u_2 + 0u_3$

Subject to $4x_1 + 3x_2 + u_1 = 12$

$$4x_1 + x_2 + u_2 = 8$$

$$4x_1 - x_2 + u_3 = 8$$

$$x_1, x_2, u_1, u_2, u_3 \geq 0$$

is the standard form of LPP

Set the values of $x_1, x_2 = 0$ then

$u_1 = 12, \; u_2 = 8$

$u_3 = 8$ is the basic solution

First Simplex Table

↓ EV

Basis	C_B	x_1	x_2	u_1	u_2	u_3	B	Ratio (θ)
u_1	0	4	3	1	0	0	12	12 / 4 = 3
u_2	0	4	1	0	1	0	8	8 / 4 =2
LV ← u_3	0	$\boxed{4}$ PE	– 1	0	0	1	8	8 / 4 = 2 ← PR
C_j		3	2	0	0	0	–	–
Z_j		0	0	0	0	0	0	–
$C_j - Z_j$		3	2	0	0	0	–	–

↑ PC

As all $C_j - Z_j$ are not ≤ 0, the solution is not optimum. Column containing x_1 is PC. As the ratio (θ) is same for rows containing u_2, u_3 any one can be considered as PR.

Let us consider the row containing u_3 as PR. Then,

Second Simplex Table (First iteration)

↓ EV

Basis	C_B	x_1	x_2	u_1	u_2	u_3	B	Ratio
u_1	0	0	4	1	0	– 1	4	4 / 4 = 1
← LV u_2	0	0	$\boxed{2}$ PE	0	1	– 1	0	0 / 2 = 0 ← PR
x_1	3	1	– 1 / 4	0	0	1/4	2	(not defined)
C_j		3	2	0	0	0	–	–
Z_j		3	– 3/4	0	0	3 / 4	6	–
$C_j - Z_j$		0	11 / 4	0	0	– 3 / 4	–	–

↑ PC

Third Simplex Table (Second iteration)

↓ EV

Basis	C_B	x_1	x_2	u_1	u_2	u_3	B	Ratio
LV ← u_1	0	0	0	1	–2	$\boxed{1}$ PE	4	4 / 1 = 4 ← PR
x_2	2	0	1	0	1 / 2	– 1 / 2	0	–
x_1	3	1	0	0	1 / 8	1 / 8	2	2 / 1 /8 = 16
C_j		3	2	0	0	0	–	–
Z_j		3	2	0	11 / 8	– 5 / 8	6	–
$C_j - Z_j$		0	0	0	– 11/ 8	5 / 8	–	–

↑ PC

Fourth Simplex Table (Third iteration)

Basis	C_B	x_1	x_2	u_1	u_2	u_3	B
u_3	0	0	0	1	– 2	1	4
x_2	2	0	$\boxed{1}$ PE	1/2	– 1/2	0	2
x_1	3	1	0	–1/8	3/8	0	3/2
C_j	–	3	2	0	0	0	–
Z_j	–	3	2	5/8	1/8	0	17/2
$C_j - Z_j$	–	0	0	–5/8	– 1/8	0	–

As all $C_i - Z_j \leq 0$ the solution is optimum

Hence, $x_1 = 3/2$, $x_2 = 2$

$Z_{max} = 17/2$

10. *Employ simplex method to solve the following LP problem*

Maximize $Z = 2x_1 + 4x_2 + 3x_3$

Subject to

$3x_1 + 4x_2 + 2x_3 \leq 60$

$2x_1 + x_2 + 2x_3 \leq 40$

$x_1 + 3x_2 + 2x_3 \leq 80$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

Solution:

First let us write down the given problem in standard form by rewriting the given constraints in the 'equations form' as follows

$3x_1 + 4x_2 + 2x_3 + u_1 = 60$, $2x_1 + x_2 + 2x_3 + u_2 = 40$

$x_1 + 3x_2 + 2x_3 + u_3 = 80$

Here u_1, u_2, u_3 are slack variables.

$u_1 = 60$, $u_2 = 40$, $u_3 = 80$

First Simplex Table

EV↓

Basis	C_B	x_1	x_2	x_3	u_1	u_2	u_3	B	Ratio
LV ← u_1	0	3	$\boxed{1}$ PE	2	1	0	0	60	60 / 4 ← PR
u_2	0	2	1	2	0	1	0	40	40 / 1
u_3	0	1	3	2	0	0	1	80	80 / 3
C_j		2	4	3	0	0	0	–	
Z_j		0	0	0	0	0	0	0	
$C_j - Z_j$		2	4	3	0	0	0	–	

↑

PC

We observe that x_2 column is PC, row is PR and is PE. Therefore, u_1 is the departing variable and x_2 is the arriving variable so that the basis for the next stage is (x_2, u_2, u_3).

Second Simplex Table

EV↓

Basis	C_B	x_1	x_2	x_3	u_1	u_2	u_3	B	Ratio
x_2	4	3 / 4	1	2 / 4	1 / 4	0	0	60/4	60 / 2
LV ← u_2	0	5 / 4	0	$\boxed{3/2}$ PE	– 1/4	1	0	25	50 / 3 ← PR
u_3	0	– 5/4	0	1 / 2	– 3/4	0	1	35	70
C_j		2	4	3	0	0	0	–	
Z_j		3	4	2	1	0	0	60	
$C_j - Z_j$		–1	0	1	– 1	0	0	–	

↑

PC

We observe that x_3 column is PC, u_2 row is PR and 3/2 is PE. Therefore, u_2 is the departing variable and x_3 is the arriving variable, so that (x_2, x_2, u_3) is the basis for the next stage.

We prepare the simplex table, as shown below

Third Simplex Table

Basis	C_B	x_1	x_2	x_3	u_1	u_2	u_3	B
x_2	4	1 / 3	0	0	1 / 3	– 1 / 3	0	20 / 3
x_3	3	5 / 6	0	1	– 1 / 6	2 / 3	0	50 / 3
u_3	0	– 5 / 3	0	0	– 2 / 3	– 1 / 3	1	80 / 3
C_j		2	4	3	0	0	0	–
Z_j		23 / 6	4	0	5 / 6	– 2 / 3	0	230 / 3
$C_j – Z_j$		– 11 / 6	0	0	– 5 / 6	– 2 / 3	0	–

We observe that in the above table, none of the entries in (C_j –Z_j) row are ≤ 0. We there fore stop the process. We find that the entries in B column corresponding to x_2, x_3 and s are 20/3, 50/3 and 230/3 respectively.

Accordingly, we conclude that x_2 = 20/3 and x_3 = 50/3 correspond to the maximum value of Z with

$$\text{Max } Z = \frac{230}{3}$$

Thus, the values of the decision variables are, $x_1 = 0$, $x_2 = 20/3$, $x_3 = 50/3$.

11. *A milk distributor supplies milk in bottles to houses in three areas A, B and C in a city. His delivery charges per bottle is 30 paise in area A, 40 paise in area B and 50 paise in area C. He has to spend on an average one minute to supply one bottle in area A, two minutes per bottle in area B and three minutes per bottle in area C. He can spare only 2½ hours for this milk distribution but not more than 1½ hours for areas A and B together. The maximum number of bottles he can deliver is 120. Find the number of bottles that he has to supply in each area so as to earn the maximum income. What is his maximum income?*

Solution:

Let the number of bottles of milk which the distributor supplies be x_1 in area A, x_2 in area B .

x_3 in area C.

and then, his total income in rupees is.

$Z = 0.3\ x_1 + 0.4x_2 + 0.5x_3$

Since he cannot supply more than 120 bottles,

$x_1 + x_2 + x_3 \leq 120$

As he requires 1 minute per bottle in area A,

2 minutes per bottle in area B,

3 minutes per bottle in area C

and he cannot spend more than 150 minutes for the work,

$x_1 + 2x_2 + 3x_3 \leq 150$

Further, as the milk distributor cannot spend more than 90 minutes for areas A and B,

$x_1 + 2x_2 \leq 90$

Obviously

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

Converting the constraints as equations.

$x_1 + x_2 + x_3 + u_1 = 120$

$x_1 + 2x_2 + 3x_3 + u_2 = 150$

$x_1 + 2x_2 + u_3 = 90$

Here, u_1, u_2, u_3 are slack variables.

Setting $x_1 = 0$, $x_2 = 0$, $x_3 = 0$ in the above equations,

we find the initial basic solution as

$u_1 = 120,\ u_2 = 150,\ u_3 = 90$

The starting and succeeding simplex tables are shown in the combined form.

		EV2↓		EV1↓					
Basis	C_B	x_1	x_2	x_3	u_1	u_2	u_3	B	**Ratio**
u_1	0	1	1	1	1	0	0	120	120/1 = 120
LV ← u_2	0	1	2	$\boxed{3}$ PE	0	1	0	150	150/3 = 50 ← PR
u_3	0	1	2	0	0	0	1	90	–
C_j		3/10	2 / 5	1/2	0	0	0	–	
Z_j		0	0	0	0	0	0	0	
$C_j - Z_j$		3/10	2 / 5	1/2	0	0	0	–	
u_1	0	2 / 3	1 / 3	0	1	– 1/3	0	70	105
x_3	1 / 2	1/3	2 / 3	1	0	1/3	0	50	150
LV ← u_3	0	1	2	0	0	0	1	90	90 ←PR
C_j		3/10	2 / 5	1/2	0	0	0	–	
Z_j		1 / 6	1 / 3	1/2	0	1/6	0	50/2	
$C_j - Z_j$		**2/15**	1/ 15	0	0	– 1/6	0	–	
u_1	0	0	– 1	0	1	– 1/3	– 2/3	10	
x_3	1 / 2	0	0	1	0	1/3	– 1/3	20	
x_1	3 / 10	1	2	0	0	0	1	90	
C_j		3/10	2 / 5	1/2	0	0	0	–	
Z_j		3/10	4 / 5	1/2	0	1/6	4/5	370	
$C_j - Z_j$		0	– 2/5	0	0	– 1/6	– 4/5	–	

From the table shown below, we note that $x_3 = 20$, $x_1 = 90$ correspond to the maximum value $Z_j = 370$. The corresponding value of x_2 is zero.

Thus, the distributor has to supply 90 bottles in area A, no bottles in area B and 20 bottles in area C so that his income is maximum, the maximum income is ₹ 370.

12. *Solve the problem*

Maximize $Z = 5x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$

$x_1 \geq 0, x_2 \geq 0$ *by using simplex method.*

Solution:

In the 'equation form', the given constraints can be written as

$x_1 + x_2 + u_1 = 2$

$5x_1 + 2x_2 + u_2 = 10$

$3x_1 + 8x_2 + u_3 = 12$

Here, u_1, u_2, u_3 are slack variables.

The constraints consists of three equations and five variables x_1, x_2, u_1, u_2, u_3 setting x_1, $x_2 = 0$, in these equations.

We get the starting basic solution

$u_1 = 2$, $u_2 = 10$, $u_3 = 12$.

First Simplex Table

EV↓

Basis	C_B	x_1	x_2	u_1	u_2	u_3	B	*Ratio*
u_1	0	1	1	1	0	0	2	2/1 = 2
LV ← u_2	0	$\boxed{5}$ PE	2	0	1	0	10	10 / 5 = 2 ← PR
u_3	0	3	8	0	0	1	12	12 /4=3
C_j		5	3	0	0	0	–	
Z_j		0	0	0	0	0	0	
$C_j - Z_j$		5	3	0	0	0	–	

↑

PC

From the table, we find that the minimum value of θ namely 2, appears in two rows the first and the second row.

Therefore, the PR is not uniquely determined. We may choose either the first row or the second row as the PR. Let us choose the second row as the PR. Then, 5 is the PE, u_2 is the departing variable and x_1 is the arriving variable.

EV↓

Basis	C_B	x_1	x_2	u_1	u_2	u_3	B	Ratio
LV ← u_1	0	0	$\boxed{3/2}$ PE	1	$-1/5$	0	0	0 ← PR
x_1	5	1	$2/5$	0	$1/5$	0	2	5
u_3	0	0	$34/5$	0	$-3/5$	1	6	$30/34$
C_j		5	3	0	0	0	–	
Z_j		5	2	0	1	0	10	
$C_j - Z_j$		0	1	0	-1	0	–	
x_2	3	0	1	$5/3$	$-1/3$	0	0	
x_1	5	1	0	$-2/3$	$1/3$	0	2	
u_3	0	0	0	$-34/3$	$5/3$	1	6	
C_j		5	3	0	0	0	–	
Z_j		5	3	$5/3$	$2/3$	0	10	
$C_j - Z_j$		0	0	$-5/3$	$-2/3$	0	–	

The succeeding simplex tables are now prepared as usual. They are shown below in the combined form.

From the table below, we find that $x_1 = 2$, $x_2 = 0$ correspond to the maximum value $Z_j = 10$ of the objective function Z. Thus, for the given problem, $x_1 = 2$, $x_2 = 0$ is the optimal basic solution with max $Z = 10$.

2.6 SPECIAL CASES OF SIMPLEX METHOD

Various special cases which may arise during the application of simplex method as discussed (like in graphical method) are summarised below.

2.6.1 Unbounded Solution

If all the ratios (θ's) are non positive. We conclude that the problem has an unbounded solution and no further working process is required.

In some cases if the value of a variable is increased indefinitely, the constraints are not violated. This indicates that the feasible region is unbounded at least in one direction.

Therefore, the objective function value can be increased indefinitely. This means that the problem has been poorly formulated or conceived.

In simplex method, this can be noticed if $C_j - Z_j$ value is positive to a variable (entering) which is notified as key column and the ratio of solution value to key column value (θ) is either negative or infinity (both are to be ignored) to all the variables. This indicates that no variable is ready to leave the basis, though a variable is ready to enter. We cannot proceed further and the solution is unbounded or not finite.

13. $Z_{max} = 5x_1 + 6x_2 + x_3$

Subject to $9x_1 - 3x_2 - 2x_3 \le 5$

$4x_1 - 2x_2 - x_3 \le 2$

$x_1 - 4x_2 + x_3 \le 3$ and $x_1, x_2, x_3 \ge 0$

Solution:

Introducing slack variables u_1, u_2 and u_3, the problem can be represented in the standard form as follows:

$$Z_{max} = 5x_1 + 6x_2 + x_3 + 0u_1 + 0u_2 + 0u_3$$

Subject to
$$9x_1 - 3x_2 - 2x_3 + u_1 = 5$$
$$4x_1 - 2x_2 - x_3 + u_2 = 2$$
$$x_1 - 4x_2 + x_3 + u_3 = 3$$
$$x_1, x_2, x_3, u_1, u_2, u_3 \ge 0$$

Simplex Table – 1

Basis	c_B	x_1	x_2	x_3	u_1	u_2	u_3	B	$\theta = B/PC$
u_1	0	9	–3	–2	1	0	0	5	–
u_2	0	4	–2	–1	0	1	0	2	–
u_3	0	1	–4	1	0	0	1	3	–
C_j		5	6	1	0	0	0	–	–
Z_j		0	0	0	0	0	0	–	
$C_j - Z_j$		5	6	1	0	0	0	–	

↑
PC

We notice that all the ratios are negative and hence are un-defined. The given solution is therefore un-bounded.

Note:

i. *As discussed in simplex method production, the ratio is defined only when it is positive. Negative/infinite values of the ratio (θ) are not considered.*

ii *This situation/case may arise in the simplex table - 1 itself or may also arise in the consequent simplex tables.*

2.6.2 Tie Breaking in Simplex Method (Degeneracy)

A condition that occurs when the number of non–zero variables in the optimal solution is less than the number of constraints is known as degeneracy. Degeneracy in LPP may arise.

i. ***At the initial stage.***

ii. ***At any subsequent iteration stages.***

iii. ***At least one of the basic variable should be zero in the initial basic feasible solution.***

In case

At any iteration of the simplex method more than one variable is eligible to leave the basis, and hence next simplex iteration produces a degenerate solution in which at least one basic variable is zero i.e., the subsequent iteration may not produce improvements in the value of the objective function. This concept is known as cycling (tie)

Procedure to Resolve Degeneracy

Step 1

Find out the rows for which the minimum non negative ratio is the same (tie), suppose there is a tie between first and third row

Step 2

Rearrange the columns of the usual simplex table so that the columns forming the original unit matrix come first in proper order

Step 3

Find the minimum of the ratio

$$\left(\frac{\text{Elements of the first column of the unit matrix}}{\text{corresponding elemens of key column}}\right)$$

Only for the tied rows

That is for the first and third rows

a. *If the third row has the minimum ratio then this ratio will be the key row*

b. *If this minimum ratio also is not unique, then go to next step*

Step 4

Find the minimum ratio, for the tied rows. If this minimum ratio is unique for the first row, then it will be the key row for determining the key element.

$$\left(\frac{\text{Elements of the second column of the unit matrix}}{\text{corresponding elemens of key column}}\right)$$

Repeat the above procedure till the minimum ratio is obtained as to resolve the degeneracy.

14. *Maximize* $Z = 3x_1 + 9x_2$

Subject to $x_1 + 4x_2 \leq 8$, $x_1 + 2x_2 \leq 4$, $x_1, x_2 \geq 0$

Solution:

Adding the slack variables we get,

Maximize $Z = 3x_1 + 9x_2\ 0u_1 + 0u_2$

Subject to $x_1 + 4x_2 + u_1 = 8$

$x_1 + 2x_2 + u_2 = 4$

First Simplex Table

↓ EV

Basis	C_B	x_1	x_2	u_1	u_2	B	Ratio
u_1	0	1	4	1	0	8	2
LV ← u_2	0	1	$\boxed{2}$ PE	0	1	4	2 ← PR
C_j		3	9	0	0		
Z_j		0	0	0	0	0	
$C_j - Z_j$		3	9	0	0		

↑ Pe ↑

Min ratio (θ) is same for both u_1, u_2

So, Find

$$\text{Min}\left(\frac{\text{elements of the first column of unit matrix}}{\text{corresponding elemens of pivot column}}\right) = \text{Min}\left(\frac{1}{4}, \frac{0}{2}\right) = 0$$

Hence u_2 is the LV thus the degeneracy is resolved and can be proceeded further as usual.

Basis	C_B	x_1	x_2	u_1	u_2	B
u_1	0	1	0	–1	2	0
x_2	9	1/2	1	0	1/2	2
C_j		3	9	0	0	
Z_j		9/2	9	0	9/2	18
$C_j - Z_j$		–3/2	0	0	–9/2	

From the above table as all $C_j - Z_j \le 0$, the solution is optimal with

$x_1 = 0$, $x_2 = 2$ and $Z_{max} = 3(0) + 9(2) = 18$.

2.6.3 Multiple optimal solutions

This situation occurs when there can be infinite number of solutions possible for a given problem. This situation can be recognised in a simplex method when one of the non-basic variables in net evaluation row in the final simplex table (optimal stage) to a problem will have a value of zero.

15. *Solve* $Z_{max} = 3x_1 + 2x_2$

Subject to

$-x_1 + 2x_2 \le 4$

$3x_1 + 2x_2 \le 14$

$x_1 - x_2 \le 3$

Solution:

Introduction slack variables u_1, u_2, u_3, the LPP can be written as,

$$Z_{max} = 3x_1 + 2x_2 + 0u_1 + 0u_2 + 0u_3$$

Subject to

$$-x_1 + 2x_2 + u_1 = 4$$

$$3x_1 + 2x_2 + u_2 = 14$$

$$x_1 - x_2 + u_3 = 3.$$

Simplex Table 1

EV ↓

Basis	c_B	x_1	x_2	u_1	u_2	u_3	B	*Ratio (B/PC)*
u_1	0	–1	2	1	0	0	4	–
u_2	0	3	2	0	1	0	14	14/3
LV ← u_3	0	$\boxed{1}$	–1	0	0	1	3	3/1 ← *PR*
C_j		3	2	0	0	0	–	
Z_j		0	0	0	0	0	0	
$C_j - Z_j$		3	2	0	0	0		

↑ *PC*

Replacing u_1 with x_1 and continuing we get,

Simplex Table (Final / optimal table) after 2 iterations.

Basis	c_B	x_1	x_2	u_1	u_2	u_3	*B*
u_1	0	0	0	1	–1/5	8/5	6
x_2	2	0	1	0	1/5	–3/5	1
x_1	3	1	0	0	1/5	2/5	4
C_j		3	2	0	0	0	–
Z_j		3	2	0	1	0	
$C_j - Z_j$		0	0	0	–1	0	

The optimal solution is: $x_1 = 4$, $x_2 = 1$, $u_1 = 6$ and $Z_{max} = 14$.

Alternate solution: Now, in the above table the non-basic variable u_3 has it's value equal to 0. This means that any increase in u_3 will bring no change.

In other words u_3 can be made a basic variable and the resulting solution will have the objective function value equal to 14. In other words, an alternate optimal solution to this problem exists which can be obtained by making u_3 as a basic variable (instead u_1) as shown in the following table.

Basis	c_B	x_1	x_2	u_1	u_2	u_3	*B*
u_3	0	0	0	5/8	–1/8	1	15/4
x_2	2	0	1	3/8	1/8	0	13/4
x_1	3	1	0	–1/4	1/4	0	5/2
C_j		3	2	0	0	0	–
Z_j		3	2	0	1	0	
$C_j - Z_j$		0	0	0	–1	0	

Thus, the alternative optimal solution is,

$$x_1 = \frac{5}{2},\ x_2 = \frac{13}{4} \qquad \text{with } Z_{max} = 14.$$

2.6.4 Infeasible Solution

Infeasibility appears when there is no solution that satisfies all of the constraints in LP problem. It is concluded that, the solution is infeasible when all $C_j - Z_j \leq 0$(optimal) and one or more artificial variable appears in the basis with positive value. When an infeasible solution exists, the LP model should be reformulated. This may be due to the fact that the model is either improperly formulated or two/more of the constraints are incompatable.

16. *Solve the following LPP*

$$Z_{max} = 6x_1 + 4x_2 \text{ subject to}$$

$$x_1 + x_2 \leq 5,\ x_2 \geq 5 \text{ and } x_1, x_2 \geq 0$$

Solution:

By adding the slack, surplus and artificial variables:

$$Z_{max} = 6x_1 + 4x_2 + 0u_1 + 0s_1 - Ma_1$$

Subject to
$$x_1 + x_2 + u_1 = 5$$
$$x_2 - s_2 + a_1 = 8$$
$$x_1, x_2, s_1, s_2, a_1 \geq 0$$

Simplex Table – 1

EV
↓

	Basis	c_B	x_1	x_2	u_1	s_1	a_1	B	θ	
LV←	u_1	6	1	$\boxed{1}$	1	0	0	5	5	←*PR*
	a_1	$-M$	0	1	0	-1	1	8	8	
	C_j		6	4	0	0	$-M$			
	Z_j		6	$6-M$	0	M	$-M$			
	$C_j - Z_j$		0	$M-2$	0	$-M$	0			

↑
PC

The variable x_2 enters and u_1 leaves and the new Simplex Table will be,

Basis	c_B	x_1	x_2	u_1	s_1	a_1	B
x_2	4	1	1	1	0	0	5
a_1	$-M$	-1	0	-1	-1	1	3
C_j		6	4	0	0	$-M$	
Z_j		$4+M$	4	$4+M$	M	$-M$	
$C_j - Z_j$		$2-M$	0	$-4-M$	$-M$	0	

Since all $C_j - Z_j \leq 0$, the solution is optimal. But the solution is not feasible for the given problem as per the constraints since it has $x_1 = 0$ and $x_2 = 5$ (it is given that $x_2 \geq 8$).

It is due to the fact that artificial variables a_1 exists in the basis with positive value($a_1 = 3$).

As the obtained solution, (the values of x_1 and x_2) is not satisfying both the constraints the solution is said to be infeasible.

2.7 ARTIFICIAL VARIABLE TECHNIQUES

So far we have discussed problems with only ≤ constraints. In general the constraints may have ≥ and = signs, after ensuring that all $b_i \geq 0$, are discussed in this chapter. In such cases basis matrix cannot be obtained as an identity matrix in the starting simplex table, therefore we introduce a new type of variable called the artificial variable along with the surplus variables. These variables are functions and cannot have any physical meaning. The artificial variable technique is merely a device / concept to get the starting Basic Feasible Solution (BFS) so that simplex method may be adapted as usual until the optimal solution is obtained.

Artificial variables are forced to be out or at zero level when the optimum solution is obtained, in other words, to get back to the original problem, artificial variables must be driven to zero in the final solution.

Two important and closely related methods are discussed here to solve the problems, with all types of constraints. They are, Big–M method and Two phase method respectively.

2.7.1 Big-M Method (Penalty Method)

Big-M (Charle's) method is a method of removing artificial variables from the basis. In this method, we assign a large undesirable (unacceptable penalty) co–efficient (– M) to artificial variable in the objective function of the standard form. The artificial variables added corresponding to ≥ and = symbols are eliminated to obtain the optimum solution.

Steps in Big-M Method

i. *Express the problem in the standard form.*

ii. *If the constraints are ≤ add slack variables only on the left hand side of the constraints*

iii. *If the constraints are ≥ subtract the surplus variables and add the artificial variables on the left–hand side of the constraints.*

iv. *If the constraints are =, add artificial variables only on left hand side of the constraints.*

v. *The coefficients of the artificial variables in the objective function are– M, M being a very big value.*

vi. *In the basis column write only the slack and artificial variables.*

vii. *Solve the problem using simplex method as usual eliminating the artificial variables.*

Outline of the Procedure/steps

SI: *Convert the LPP into equation form by introducing the necessary slack and surplusvariables.*

S2: *Introduce non-negative variables to the left-hand side of all the constraints of greater than or equal type. These variables are called artificial variables. The purpose of introducing artificial variables is just to obtain an initial basic feasible solution. In order to get rid of the artificial variables in the final optimum iteration, we assign a very large penaulty –M, in maximization problems to the artificial variables in the objective function.*

S3: Solve the modified LPP by simplex method.

Whenver an artificial variable happens to leave the basis, we drop articial variable and omot all the entries corresponding to it's column from the simplex table.

S4: Application of simplex method is continued until, either an optimum basic feasible solution is obtained or there is an indication of the existence of an unbounded solution to the given LPP.

Note: *While making iterations by this method, one of the following three cases may arise.*

i. *If no artificial variable remains in the basis and the optimal condition is satisfied, then the current solution is an optimal basic feasible solution.*

ii. *If atleast one artificial variable appears in the basis at zero level and the optimality condition is satisfied, then the current solution is anoptimal basic feasible solution.*

iii. *If atleat one b0asic variable appears in the basis of non-zero level and the optimality condition is satisfied, then the original problem has no fasible solution.*

2.7.2 Two Phase Method

Linear programming problems having the constraints as ≥, = signs can also be solved by using two phases.

In the first phase, artificial variables are eliminated from the simplex table and in the second phase using the solution obtained in first phase, optimal solution is obtained. Since, the optimal solution is obtained in two phases; it is called "Two Phase Simplex Method."

Steps in Two Phase Method

Phase-1

Construct an auxiliary LPP

Auxiliary LPP is one, in which objective function is with zero coefficients for slack, basic variables and a – 1 coefficient for artificial variables. Apply the simplex method, and solve the auxiliary LPP until either of the cases arise.

i. Cj – Zi ≤ 0 and even if and at least one artificial variable appears in the basis row at a positive level, then the problem does not posses any feasible solution. No need to proceed further

ii. Cj – Zi ≤ 0 and no artificial variable exists in the basis row and if artificial variable exists but it takes zero value then go to phase-2.

Phase-2

Modify the objective function considering the actual coefficients to the decision variables, '0' coefficients to slack, surplus variable get the modified / actual objective function (do not include artificial variable).

Apply the simplex method and solve as usual until optimal solution is obtained.

Note:

i. *If the constraint is ≤, add only a slack variable.*

ii. *If the constraint is ≥, subtract the surplus variable and add an artificial variable.*

iii. *If the constraint is =, add only artificial variable.*

Let us understand these two methods through the following worked examples.

17. *Max* $Z = 2x_1 + 3x_2$ *subject to* $x_1 + 2x_2 \le 4$ *and* $x_1 + x_2 = 3$
$x_1 \ge 0, x_2 \ge 0$ *using Big-M method*

Solution:

$x_1 + 2x_2 + u_1 = 4$

$x_1 + x_2 + a_1 = 3$

$Z_{max} = 2x_1 + 3x_2 + 0u_1 - Ma_1$

First Simplex Table

EV ↓

Basis	C_B	x_1	x_2	u_1	a_1	B	θ
LV ← u_1	0	1	$\boxed{2}$ PE	1	0	4	2 ← PR
a_1	– M	1	1	0	1	3	3
C_j		2	3	0	– M	–	–
Z_j		– M	– M	0	– M	– 3M	–
$C_j - Z_j$		2 +M	3 + M	0	0	–	–

↑ PC

As all $C_j - Z_j$ are not ≤ 0, let us move to the next simplex table

Second Simplex Table (First Iteration)

EV↓

Basis	C_B	x_1	x_2	u_1	a_1	B	Ratio
x_2	3	1/2	1	1/2	0	2	4
LV ← a_2	– M	$\boxed{1/2}$ PE	0	– 1/2	1	1	2 ← PR
Z_j		– M / 2 + 3/ 2	3	M / 2 + 3/2	– M	–	–
$C_j - Z_j$		M +1 / 2	0	– M – 3 /2	0	–	–

↑

PC

Third Simplex Table (Second Iteration)

Basis	C_B	x_1	x_2	u_1	B
x_2	3	0	1	3/4	1
x_1	2	1	0	– 1	2
C_j		2	3	0	–
Z_j		2	3	1 / 4	7
$C_j – Z_j$		0	0	–1 / 4	–

As all $C_j - Z_j \leq 0$, the solution is optimum

Hence, $x_1 = 2$, $x_2 = 1$

$Z_{max} = 2\ (2) + 3\ (1) = 7$

18. *Use Penalty (or Big-M) Method to*
Maximize $Z = 6x_1 + 4x_2$ *subject to the constraints:*
$2x_1 + 3x_2 \leq 30,\ 3x_1 + 2x_2 \leq 24,\ x_1 + x_2 \geq 3$ *and* $x_1, x_2 \geq 0$

Solution:

Converting the inequalities into equations by adding slack, surplus and / or artificial variables we get,

$$2x_1 + 3x_2 + u_1 = 30$$

$$3x_1 + 2x_2 + u_2 = 24$$

$$x_1 + x_2 - s_1 + a_1 = 3$$

The modified objective function (considering – M) is,

$Z_{max} = 6x_1 + 4x_2 + 0u_1 + 0u_2 + 0s_1 - Ma_1$

Initial basic feasible solution is

$u_1 = 30$ $u_2 = 24$ and $a_1 = 3$ [setting x_1, x_2, $s_1 = 0$]

First Simplex Table

EV↓

Basis	C_B	x_1	x_2	u_1	u_2	s_1	a_1	B	Ratio
u_1	0	2	3	1	0	0	0	30	15
u_2	0	3	2	0	1	0	0	24	8
LV ← a_1	– M	$\boxed{1}$ PE	1	0	0	– 1	1	3	3 ← PR
C_j		6	4	0	0	0	– M	–	–
Z_j		– M	– M	0	0	M	– M	–	–
$C_j – Z_j$		6 + M	4 + M	0	0	– M	0	–	–

↑

PC

As all $C_j - Z_j$ are not lesser than or equal to zero the solution is not optimum. Hence, move to the next table revising the basis keeping the PE as unity, making other elements in PC as zeros.

Second Simplex Table (First iteration)

(Replace a_1 by x_1, keep the pivot element unity (1) and make the other two elements in the pivotal element column as zeros)

EV↓

Basis	C_B	x_1	x_2	u_1	u_2	s_1	B	Ratio
u_1	0	0	1	1	0	2	24	12
LV ← u_2	0	0	– 1	0	1	$\boxed{3}$ PE	15	5 ← PR
x_1	6	1	1	0	0	– 1	3	–
C_j		6	4	0	0	0	–	–
Z_j		6	6	0	0	– 6	–	–
$C_j - Z_j$		0	– 2	0	0	6	–	–

↑PC

Third Simplex Table (Second Iteration)

Basis	C_B	x_1	x_2	u_1	u_2	s_1	B
u_1	0	0	5/3	1	–2/3	0	14
s_1	0	0	–1/3	0	1/3	1	5
x_1	6	1	2/3	0	1/3	0	8
C_j		6	4	0	0	0	–
Z_j		6	4	0	2	0	48
$C_j - Z_j$		0	0	0	– 2	0	–

As all $C_j - Z_j \leq 0$, the solution is optimum

The optimum solution is

$x_1 = 8$ and $x_2 = 0$ with max $Z = 48$

Note:

i. Consider only slack and artificial variables in the basis (C_B) of the first simplex table.

ii. Remove the artificial variable from the table whenever it becomes leaving variable in the process.

iii. It may exist in the basis with its 'B' (RHS) value equal to zero.

19. *Use Two Phase simplex method to maximize*

$Z_{max} = 5x_1 + 3x_2$

Subject to $2x_1 + x_2 \leq 1,\ x_1 + 4x_2 \geq 6$ *and* $x_1, x_2 \geq 0$

Solution:

$2x_1 + x_2 + u_1 = 1$

$x_1 + 4x_2 - s_1 + a_1 = 6$

Auxiliary L.P.P. is,

$0x_1 + 0x_2 + 0u_1 + 0s_1 - a_1$

[Auxiliary LPP is one, in which objective function is with zero co–efficients for x_1, x_2, u_1, s_1 but – 1 to a_1]

That is an objective function with zero coefficients for slack, surplus variables and a –ve coefficient (– 1) for artificial variables.

Phase-1

EV ↓

Basis	C_B	x_1	x_2	u_1	s_1	a_1	B	Ratio
LV ← u_1	0	2	$\boxed{1}$ PE	1	0	0	1	1 ← PR
a_1	– 1	1	4	0	– 1	1	6	1.5
C_j		0	0	0	0	– 1	–	–
Z_j		– 1	– 4	0	1	– 1	–	–
$C_j – Z_j$		1	4	0	– 1	0	–	–

↑

PC

u_1 will be replaced by x_2

Basis	C_B	x_1	x_2	u_1	s_1	a_1	B
x_2	0	2	1	1	0	0	1
a_1	– 1	– 7	0	– 4	– 1	1	2
C_j		0	0	0	0	– 1	–
Z_j		7	0	4	1	– 1	–
$C_j – Z_j$		– 7	0	– 4	– 1	0	–

As all $C_j – Z_j \leq 0$ an optimum basic feasible solution to the auxiliary LPP is obtained. But an artificial variable in the basis at a positive Level ($a_1 = 2$) exists. Therefore, the original LPP does not possess any feasible solution.

20. *Solve the following LPP using two-phase simplex method Minimize* $Z = 4x_1 + x_2$ *Subject to* $x_1 + 2x_2 + 6x_3 \leq 4,\ 4x_1 + 3x_2 + 2x_3 \geq 6,\ 3x_1 + x_2 = 3$ *and* $x_1, x_2 \geq 0.$

Solution

As the given objective function is of minimisation type express / convert it into maximisation form (standard form) by changing the sign of the variables. By doing so we get,

$Z_{max} = -4x_1 - x_2$

Converting the inequalities into equations by using appropriate variables we get,

$x_1 + 2x_2 + 6x_3 + u_1 = 4, \quad 4x_1 + 3x_2 + 2x_3 - s_1 + a_1 = 6$

$3x_1 + x_2 + a_2 = 3 \quad$ where, $x_1, x_2, u_1, s_1, a_1, a_2 \geq 0$

Auxiliary LPP is $Z_{max} = 0x_1 + 0x_2 + 0u_1 + 0s_1 - a_1 - a_2$

Phase-1

Simplex Table-1

EV↓

Basis	C_B	x_1	x_2	u_1	s_1	a_1	a_2	B	Ratio
u_1	0	1	2	1	0	0	0	4	4
a_1	–1	4	3	0	–1	1	0	6	1.5
LV ← a_2	–1	$\boxed{3}$ PE	1	0	0	0	1	3	1 ← PR
C_j		0	0	0	0	–1	–1	–	–
Z_j		–7	–4	0	1	–1	–1	–	–
$C_j–Z_j$		7	4	0	–1	0	0	–	–

↑

PC

Simplex Table-2

EV↓

Basis	C_B	x_1	x_2	u_1	s_1	a_1	B	Ratio
u_1	0	0	5/3	1	0	0	3	9/5
LV ← a_1	–1	0	$\boxed{5/3}$ PE	0	–1	1	2	6/5 ← PR
x_1	0	1	1/3	0	0	0	1	3
C_j		0	0	0	0	–1	–	–
Z_j		0	–5/3	0	1	–1	–	–
$C_j–Z_j$		0	5/3	0	–1	0	–	–

↑

PC

As all $C_j - Z_j$ are not less than or equal to 0, the solution is not yet optimal or phase – 1 is not yet completed.

Simplex Table-3

Basis	C_B	x_1	x_2	u_1	s_1	B
u_1	0	0	0	1	1	1
x_2	0	0	1	0	–3/5	6/5
x_1	0	1	0	0	1/5	3/5
C_j		0	0	0	0	–
Z_j		0	0	0	0	–
$C_j - Z_j$		0	0	0	0	–

Solution is optimum in phase – 1 as all $(C_j - Z_j) \leq 0$

Phase-2

Z modified/actual $= Z_{max} = -4x_1 - x_2 + 0\,u_1 + 0\,s_1$

Basis	C_B	x_1	x_2	u_1	s_1	B
u_1	0	0	0	1	1	1
x_2	–1	0	1	0	–3/5	6/5
x_1	–4	1	0	0	1/5	3/5
C_j		0	0	0	0	–
Z_j		0	0	0	0	–18/5
$C_j - Z_j$		0	0	0	0	–

The solution is optimum as, all $C_j - Z_j \leq 0$

Hence $x_1 = 3/5$, $x_2 = 6/5$

$Z_{max} = -4x_1 - x_2$ i.e $-4(3/5) - (6/5) = -18/5$

$Z_{min} = -Z_{max} = 18/5$.

21. Using two phase method solve the LPP

Maximise $Z = 7.5x_1 - 3x_2$

Subject to the conditions

$3x_1 - x_2 - x_3 \geq 3$, $x_1 - x_2 + x_3 \geq 2$ and $x_1, x_2, x_3 \geq 0$

Solution:

Given, $Z_{max} = 7.5\,x_1 - 3x_2$

Such that $3x_1 - x_2 - x_3 \geq 3$, $x_1 - x_2 + x_3 \geq 2$ and $x_1, x_2, x_3 \geq 0$

Phase – 1

Converting the in equations as equations we get,

$$3x_1 - x_2 - x_3 - s_1 + a_1 = 3$$

$$x_1 - x_2 + x_3 - s_2 + a_2 = 2$$

The auxilary LPP is

$$Z_{max} = 0x_1 + 0x_2 + 0s_1 + 0s_2 - a_1 - a_2$$

Simplex Table – 1

↓ EV

	Basis	C_B	x_1	x_2	x_3	s_1	s_2	a_1	a_2	B	Ratio	
← LV	a_1	–1	$\boxed{3}$ PE	–1	–1	–1	0	1	0	3	1	← PR
	a_2	–1	1	–1	1	0	–1	0	1	2	2	
	C_j		0	0	0	0	0	–1	–1	–		
	Z_j		–4	2	0	1	1	–1	–1		–	
	$C_j - Z_j$		4	–2	0	–1	–1	0	0		–	

↑PC

(The solution is not optimum)

Keeping the PE as 1, making other element of PC(i.e.1) as '0' ad replacing the LV with we get,

Simplex Table-2

↓ EV

	Basis	C_B	x_1	x_2	x_3	s_1	s_2	a_2	B	Ratio	
	x_1	0	1	–1/3	–1/3	–1/3	0	0	1	–	
← LV	a_2	–1	1	–2/3	$\boxed{4/3}$ PE	1/3	–1	1	1	3/4	← PR
	C_j		0	0	0	0	0	–1			
	Z_j		0	2/3	–4/3	–1/3	1	0			
	$C_j - Z_j$		0	–2/3	4/3	1/3	–1	–1		–	

↑PC

(The solution is not yet optimum as all $C_j - Z_j$ are not ≤ 0

Making the PE as '1', the other element of PC as '0' ad replacing the LV with EV we get,

Basis	C_B	x_1	x_2	x_3	s_1	s_2	B
x_1	0	1	−1/2	0	+1/4	+1/4	5/4
x_3	0	0	−2/3	1	1/4	−3/4	3/4
C_j		0	0	0	0	0	–
Z_j		0	0	0	0	0	
$C_j - Z_j$		0	0	0	0	0	

The solution is optimum for phase – 1 as all $C_j - Z_j \leq 0$

Hence, we can proceed to phase – 2

Phase-2

The actual objective function is, $Z_{max} = 7.5\ x_1 - 3x_2 + 0s_1 + 0s_2$

Simple Table

Basis	C_B	x_1	x_2	x_3	s_1	s_2	B
x_1	7.5	1	+1/2	0	+1/4	−1/4	5/4
x_3	0	0	−2/3	1	1/4	−3/4	3/4
C_j		7.5	−3	0	0	0	–
Z_j		7.5	−7.5/2	0	+7.5/4	+7.5/4	
$C_j - Z_j$		0	−3/4	0	−7.5/4	−7.5/4	–

The solution is optimum

$$x_1 = \frac{5}{4},\ x_3 = \frac{3}{4},\ x_2 = 0$$

$$Z_{max} = 7.5\left(\frac{5}{4}\right) - 3(0)$$

$$= \frac{75}{8}$$

22. *Solve the following problem.*

$$Z_{min} = 3x_1 + 2x_2 + 4x_3$$

Subject to $2x_1 + x_2 + 3x_3 = 60$ *and* $3x_1 + 3x_2 + 5x_3 \geq 120$

(a) Using Big – M Method *(b) Using Two Phase Method*

Solution:

a) **Constraints in modified form are,**

$2x_1 + x_2 + 3x_3 + a_1 = 60$

$3x_1 + 3x_2 + 5x_3 - s_1 + a_2 = 120$

Initial basic feasible solution is

$a_1 = 60$

$a_2 = 120$

Modified objective function

$Z_{max} = -3x_1 - 2x_2 - 4x_3 + 0s_1 - Ma_1 - Ma_2$

First Simplex Table

E.V ↓

Basis	C_B	x_1	x_2	x_3	s_1	a_1	a_2	B	Ratio
LV ← a_1	$-M$	2	1	$\boxed{1}$ PE	0	1	0	60	20 ← PR
a_2	$-M$	3	3	5	-1	0	1	120	24
C_j		-3	-2	-4	0	$-M$	$-M$	–	
Z_j		$-5M$	$-4M$	$-8M$	M	$-M$	$-M$	–	
$C_j - Z_j$		$5M-3$	$4M-2$	$8M-4$	$-M$	0	0	–	–

↑

PC

Second Simplex Table

E.V ↓

Basis	C_B	x_1	x_2	x_3	s_1	a_2	B	Ratio
x_3	-4	2/3	1/3	1	0	0	20	60
LV ← a_2	$-M$	$-1/3$	$\boxed{4/3}$ PE	0	-1	1	20	15 ← PR
C_j		-3	-2	-4	0	M	–	–
Z_j		$M - 8/3$	$-4M - 4/3$	-4	M	$-M$	–	–
$C_j - Z_j$		$-M - 1/3$	$4M - 2/3$	0	$-M$	0	–	–

↑

PC

Third Simplex Table

Basis	C_B	x_1	x_2	x_3	s_1	B
x_3	-4	$3/4$	0	1	$1/4$	15
x_2	-2	$-1/4$	1	0	$-3/4$	15
C_j		-3	-2	-4	0	–
Z_j		$-5/2$	-2	-4	$1/2$	–
$C_j - Z_j$		$-1/2$	0	0	$-1/2$	–

Since all $C_j - Z_j$ value is ≤ 0, the solution is optimum

Therefore $x_3 = 15$, $x_2 = 15$, $x_1 = 0$

$= 30 + 60$

$Z_{max} = 90$, $\quad Z_{min} = -(Z_{max}) = -90$

b) Two phase method

Modified constraints are,

$2x_1 + x_2 + 3x_3 + a_1 = 60$

$3x_1 + 3x_2 + 5x_3 - s_1 + a_2 = 120$

Setting the basic, slack variables to zero we get, $a_1 = 60$, $a_2 = 120$

Auxiliary LPP is, $Z_{max} = 0x_1 + 0x_2 + 0x_3 + 0s_1 - 1a_1 - 1a_2$

Phase 1

Simplex Table-1

E.V ↓

Basis	C_B	x_1	x_2	x_3	s_1	a_1	a_2	B	θ
LV ← a_1	-1	2	1	$\boxed{3}$ PE	0	1	0	60	20 ← PR
a_2	-1	3	3	5	-1	0	1	120	24
C_j		0	0	0	0	-1	-1	–	–
Z_j		-5	-4	-8	1	-1	-1	–	–
$C_j - Z_j$		5	4	8	-1	0	0	–	–

↑

PC

Simplex Table-2

E.V ↓

Basis	C_B	x_1	x_2	x_3	s_1	a_2	B	θ
x_3	0	2 / 3	1 / 3	1	0	0	20	60
LV ← a_2	– 1	– 1/ 3	PE $\boxed{4/3}$	0	– 1	1	20	15 ← PR
C_j		0	0	0	0	– 1	–	–
Z_j		1/3	– 4/3	0	1	– 1	–	–
$C_j - Z_j$		– 1/3	4/3	0	– 1	0	–	–

↑

PC

Simplex Table-3

Basis	C_B	x_1	x_2	x_3	s_1	B
x_3	0	3/4	0	1	1/4	15
x_2	0	– 1/4	1	0	– 3/4	15
C_j		0	0	0	0	–
Z_j		0	0	0	0	–
$C_j - Z_j$		0	0	0	0	–

Since all $C_j - Z_j \leq 0$, it is a basic feasible solution.

Phase 2

The actual objective function is, $Z_{max} = -3x_1 - 2x_2 - 4x_3 + 0s_1$

Basis	C_B	x_1	x_2	x_3	s_1	B
x_3	– 4	3 / 4	0	1	1 / 4	15
x_2	– 2	– 1 / 4	1	0	– 3 / 4	15
C_j		– 3	– 2	– 4	0	–
Z_j		– 5/2	– 2	– 4	1/2	–
$C_j - Z_j$		– 1/2	0	0	– 1/2	–

Since all $C_j - Z_j$ values ≤ 0, the solution is optimal.

$x_3 = 15,\ x_2 = 15,\ x_1 = 0$

$Z_{min} = 15 \times 2 + 15 \times 4$

$= 90$

23. Solve by two phase method $Z_{max} = 3x_1 + 2x_2 + 3x_3$

Subject to $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

Solution:

Modified constraints are

$2x_1 + x_2 + x_3 + u_1 = 2$

$3x_1 + 4x_2 + 2x_3 - s_1 + a_1 = 8$

$Z_{max} = 0x_1 + 0x_2 + 0x_3 + 0u_1 + 0s_1 - a_1$

Phase-1

Simplex Table–1

E.V ↓

Basis	C_B	x_1	x_2	x_3	u_1	s_1	a_1	B	Ratio
u_1	0	2	1	1	1	0	0	2	2
LV ←a_1	– 1	3	PE $\boxed{4}$	2	0	– 1	1	8	2 ← PR
C_j		0	0	0	0	0	– 1	–	–
Z_j		– 3	–4	– 2	0	1	– 1	–	–
$C_j - Z_j$		3	4	2	0	– 1	0	–	–

↑

PC

Simplex Table-2

Basis	C_B	x_1	x_2	x_3	u_1	s_1	B
u_1	0	5 / 4	0	1 / 2	1	1 / 4	0
x_2	0	3 / 4	1	1 / 2	0	– 1 / 4	2
C_j		0	0	0	0	0	–
Z_j		0	0	0	0	0	–
$C_j - Z_j$		0	0	0	0	0	–

Since all $C_j - Z_j$ values are ≤ 0, the solution is optimum in phase 1.

Phase – 2

$Z_{max} = 3x_1 + 2x_2 + 3x_3 + 0u_1 + 0s_1$

EV ↓

Basis	C_B	x_1	x_2	x_3	u_1	s_1	B	Ratio
LV ← u_1	0	5 / 4	0	PE $\boxed{1/2}$	1	1 / 4	0	0 ← PR
x_2	2	3 / 4	1	1 / 2	0	– 1 / 4	2	1
C_j		3	2	3	0	0		
Z_j		3/2	2	1	0	– 1 / 2		
$C_j - Z_j$		3/2	0	2	0	1 / 2		

↑PC

Basis	C_B	x_1	x_2	x_3	u_1	s_1	B
x_3	3	5 / 2	0	1	2	1 / 2	0
x_2	2	– 2 / 4	1	0	– 1	– 1 / 2	2
C_j		3	2	3	0	0	
Z_j		13 / 2	2	3	5	1 / 2	
$C_j - Z_j$		– 3.5	0	0	– 5	– 1 / 2	

All $C_j - Z_j$ values are ≤ 0, hence solution is optimal

$\therefore x_1 = 0, x_2 = 2, x_3 = 0$

$Z_{max} = 4$

24. *Minimize* $Z_j = 600\,x_1 + 500\,x_2$

Subject to $2x_1 + x_2 \geq 80$

$x_1 + 2x_2 \geq 60$

$x_1, x_2 \geq 0$

Solution:

By introducing surplus variables s_1, s_2 and artificial variables a_1, a_2 in the inequalities of the constraints we get,

$2x_1 + x_2 - s_1 + a_1 = 80$

$x_1 + 2x_2 - s_1 + a_2 = 60$

$Z_{max} = -\,600x_1 - 500x_1, 0s_1 + 0s_2 - Ma_1 - Ma_2$

(Objective function in standard form)

First Simplex Table

EV ↓

Basis	C_B	x_1	x_2	s_1	s_2	a_1	a_2	B	*Ratio*
a_1	$-M$	2	1	-1	0	1	0	80	80
LV ← a_2	$-M$	1	$\boxed{2}$ PE	0	-1	0	1	60	30 ← PR
C_j		-600	-500	0	0	$-M$	$-M$	–	–
Z_j		$-3M$	$-3M$	$+M$	M	$-M$	$-M$	–	–
$C_j - Z_j$		$-600 + 3M$	$-500 + 3M$	$-M$	$-M$	0	0	–	–

↑

PC

Selected pivot column is with $-500 + 3M$ or $3M - 500$ is the most positive compared to $3M - 600$, x_2 replaces a_2

$$R_2\text{ (new)} = \frac{R_2\text{(old)}}{PE},\ R_1\text{ (New)} = R_1\text{ (old)} - R_2\text{ (new)}$$

Second Simplex Table

EV ↓

Basis	C_B	x_1	x_2	s_1	s_2	a_1	B	Ratio
LV ← a_1	$-M$	$\boxed{3/2}$ PE	0	-1	$1/2$	1	50	100/3 ← PR
x_2	-500	$1/2$	1	0	$-1/2$	0	30	60
C_j	–	-600	-500	0	0	$-M$	–	–
Z_j	–	$-250 - 3M/2$	-500	M	$250 - M/2$	$-M$	–	–
$C_j - Z_j$	–	$-350 + 3M/2$	0	$-M$	$-250 + M/2$	0	–	–

↑

PC

x_1 replaces a_1

in the new table

R_1 (new) = R_1 (old) / pivot element

R_2 (new) = R_2 (old) – respective pivot column element × R_1 (new)

Third Simplex Table

Basis	C_B	x_1	x_2	s_1	s_2	B
x_1	– 600	1	0	– 2/3	1 /3	100 / 3
x_2	– 500	0	1	1/3	– 2/3	40 / 3
C_j		– 600	– 500	0	0	–
Z_j		– 600	– 500	700/3	400/3	–
$C_j - Z_j$		0	0	– 700/3	– 400/3	–

As all $C_j - Z_j \leq 0$, the Solution is optimum

$x_1 = 100 / 3$, $x_2 = 40 / 3$ with $Z_{min} = 80{,}000 / 3$

REVIEW QUESTIONS

1. *Mention the limitations of graphical method used to solve a LPP.*
2. *Define the following terms in connection with LPP.*
   i. Slack variable. ii. Surplus variable. iii. Basic solution.
   iv. Basic feasible solution. v. Optimal solution. vi. Infeasible solution
   vii. Unbounded solution
3. *Explain the essence of simplex method.*
4. *Explain the setting up of simplex method.*
5. *Why simplex method is better than graphical method?*
6. *Write a brief note on unbounded solution and infeasible solution of simplex method.*
7. *Explain the following*
   - *i.* *A standard form of the LPP*
   - *ii.* *Basic solution of an LPP*
   - *iii.* *Degeneracy and un-bounded solution with respect to simplex method.*
8. *Write procedure to solve LPP of two phase simplex method.*
   - *i.* *Un-bounded solution*
   - *ii.* *Infeasible solution*
   - *iii.* *Multiple alternative solutions*
9. *Illustrate with examples slack and surplus variables.*
10. *Give the characteristics of LPP.*
11. *Why simplex method is a better technique than graphical for most real cases ? Explain.*
12. *Explain the steps needed to find feasible solution using simplex method.*

13. *Write key solution concepts of simplex method.*
14. *What is degeneracy in LPP and how do you resolve it?*
15. *Explain the concept of tie breaking in simplex method*
16. *Give the algebra of simplex method.*
17. *Write a note on artificial variables.*
18. *Explain briefly Big – M method and Two phase method.*
19. *Write a note on auxiliary LPP.*
20. *Explain two phase technique to solve LPP in simplex method.*

PROBLEMS

1. *Find all the basic solutions to the following problems*

$Z_{max} = x_1 + 3x_2 + 3x_3$

Subject to the constraints

$x_1 + 2x_2 + 3x_3 = 4$ and $2x_1 + 3x_2 + 5x_3 = 7$

Stating when the solutions are feasible.

Ans: *x_1, x_2 are basic, x_3 is non basic variables $z_{max} = 5$ (Feasible)*

x_1, x_3 are basic, x_2 is non basic variables $z_{max} = 4$ (Feasible)

x_2, x_3 are basic, x_1 is non basic variables $z_{max} = 3$ (Solution is not feasible)

2. For the following system of liner equations.

$2x_1 + 3x_2 + 4x_3 = 10$ $3x_1 + 4x_2 + x_3 = 12$

classify the solutions in to

i) Basic feasible solution.

ii) Degenerate basic feasible solution

iii) Non degenerate basic feasible solution

Ans: *i) Basic Feasible Solutions: a) when $x_1 = 0, x_2 = 38/13, x_3 = 4/13$*

b) when $x_2 = 0, x_1 = 19/5, x_3 = 3/5$

ii) There are no degenerate basic solutions

iii) Non degenerate basic feasible solutions: a) when $x_1 = 0, x_2 = 38/13, x_3 = 4/13$

b) when $x_2 = 0, x_1 = 19/5, x_3 = 3/5$

3. *Solve the following LPP by using simplex method*

$Z_{max} = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \leq 40$

$x_1 - x_2 \le 20$ and $x_1, x_2 \ge 0$

Ans: $x_1 = 30,\ x_2 = 10,\ Z_{max} = 110$

4. *Max* $Z = x_1 + x_2 + 3x_3$ *by using simplex method*

Subject to $3x_1 + 2x_2 + x_3 \le 3$

$2x_1 + x_2 + 2x_3 \le 2$ and $x_1, x_2, x_3 \ge 0$

Ans: $x_1 = 0,\ x_2 = 0,\ x_3 = 1$ *and* $Z_{min} = 3$

5. *Min* $Z = x_1 - 3x_2 + 2x_3$ *by using simplex method*

Subject to $3x_1 - x_2 + 2x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$ and $x_1, x_2, x_3 \ge 0$

Ans: $x_1 = 4,\ x_2 = 5,\ x_3 = 0$ *and* $Z_{min} = -11$

6. *Max* $Z = 3x_1 + 4x_2$ *by using simplex method*

Subject to $x_1 - x_2 \le 1$

$-x_1 + x_2 \le 2$ and $x_1, x_2, \ge 0$

Ans: *The solution is unbounded*

7. *Max* $Z = 40x + 25y + 50z$ *by using simplex method*

Subject to $x + 2y + z \le 36$

$2x + y + 4z \le 60,$

$2x + 5y + z \le 45$ and $x, y, z \ge 0$

Ans: $x = 20,\ y = 0,\ z = 5$ *and* $Z_{min} = 1050$

8. *Solve by using simplex method*

$Z_{max} = 10x_1 + 6x_2$

Subject to $x_1 + x_2 \le 2,\ 2x_1 + x_2 \le 4$

$3x_1 + 8x_2 \le 12$ and $x_1, x_2 \ge 0$

Ans: $x_1 = 2,\ x_2 = 0$ *and* $Z_{max} = 20$

9. *Solve the following LPP by simplex method*

$Z_{max} = 10x_1 + 15x_2 + 8x_3$

Subject to $x_1 + 2x_2 + 2x_3 \le 200$

$2x_1 + x_2 + x_3 \le 220$

$3x_1 + x_2 + 2x_3 \le 180$

$x_1 \geq 10$, $x_2 \geq 20$

$x_3 \geq 30$ and $x_1, x_2, x_3 \geq 0$

Ans: *$x_1 = 20$, $x_2 = 60$, $x_3 = 30$ and $Z_{max} = 1340$*

10. Solve by simplex method

$Z_{max} = 2x_1 + 3x_2$

Subject to the constraints $x_1 \leq 5$, $x_2 \geq 10$ and $x_1, x_2 \geq 0$

Ans: ***Solution is infeasible***

11. $Z_{max} = 4x_1 + 10x_2$

Subject to the constraints

$2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$ and $2x_1 + 3x_2 \leq 90$

Ans: *$x_1 = 0$, $x_2 = 20$ and $Z_{max} = 200$*

12. $Z_{max} = 5x_1 + 8x_2$

Subject to the constraints $3x_1 + 2x_2 \leq 36$

$x_1 + 2x_2 \leq 20$ and $3x_1 + 4x_2 \leq 42$

Ans: *$x_1 = 2$, $x_2 = 9$ and $Z_{max} = 82$*

13. $Z_{max} = 3x_1 + 4x_2$

Subject to the constraints $x_1 + x_2 \leq 45$

$2x_1 + x_2 \leq 60$ and $x_1, x_2 \geq 0$

Ans: *$x_1 = 0$, $x_2 = 45$ and $Z_{max} = 180$*

14. $Z_{max} = 3x_1 + 5x_2$

Subject to the constraints $x_1 \leq 4$, $2x_2 \leq 12$, $3x_1 + 2x_2 \leq 18$ and $x_1, x_2 \geq 0$

Ans: *$x_1 = 2$, $x_2 = 6$ and $Z_{max} = 36$*

15. $Z_{max} = 5x_1 + 4x_2$

Subject to $6x_1 + 4x_2 \leq 24$

$x_1 + 2x_2 \leq 6$

$-x_1 + x_2 \leq 1$

Ans: *$x_1 = 3$, $x_2 = 0.5$ and $Z_{max} = 21$*

16. Solve $Z_{max} = 20x_1 + 24x_2$

Subject to: $2x_1 + 3x_2 \leq 150$

$3x_1 + 2x_2 \leq 150$

$x_2 \leq 450$

$x_1, x_2 \geq 0.$

Ans: $x_1 = 30, x_2 = 30$ *and* $Z_{max} = 1320$

17. ***Using Big – M method solve the LPP***

Maximize $Z = 2x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$ and $x_1, x_2 \geq 0$

Ans: $x_1 = \frac{3}{5}, \quad x_2 = \frac{6}{5},$ and $z_{max} = \frac{12}{5}$

18. ***Use Big – M method to solve***

$Z_{min} = 4x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$ and $x_1, x_2 \geq 0$

Ans: $x_1 = 0, x_2 = -3,$ *and* $Z_{min} = 3$

19. ***Solve the following LPP by two – phase simplex method***

$Z_{max} = 3x_1 - x_2$

Such that $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 2, x_2 \leq 4$ and $x_1, x_2 \geq 0$

Ans: $x_1 = 2, x_2 = 0$ *and* $Z_{max} = 6$

20. ***Solve the following LPP either by Big M method or two phase method***

$Z_{max} = 2x_1 + x_2 + 3x_2$

Subject to $x_1 + x_2 + 2x_3 \leq 5$

$2x_1 + 3x_2 + 4x_3 = 12$ and $x_1, x_2, x_3 \geq 0$

Ans: $x_1 = 3, x_2 = 2, x_3 = 0, Z_{max} = 8$

21. ***Using two phase method solve the following LPP***

$Z_{min} = -2x_1 - x_2$

Subject to $x_1 + x_2 \geq 2$

$x_1 + x_2 \leq 4$ and $x_1, x_2 \geq 0$

Ans: $Z_{min} = -8, x_1 = 4, x_2 = 0$

22. *Solve the following problem either by Big – M method or by dual simplex method.*

$Z_{min} = 2x_1 + 2x_2 + 4x_3$

Subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$ and $x_1\ x_2, x_3 \geq 0$

Ans: $x_1 = 0, \quad x_2 = \frac{2}{3}, x_3 = 0$ and $Z_{min} = \frac{4}{3}$

23. *Solve the following LPP by simplex method.*

$Z_{max} = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 + 5x_3 \leq 2$

$3x_1 + 4\ x_2 \geq 12$ and $x_1, x_2 \geq 0$

Ans: *Solution is infeasible*

24. *Using Big M method solve,*

$Z_{min} = 4x_1 + 3x_2$

Subject to $2x_1 + x_2 \geq 10$

$-3x_1 + 2\ x_2 \leq 6$ and $x_1 + x_2 \geq 6$

Ans: $x_1 = 4, x_2 = 2$ *and* $Z_{min} = 22$

25. ✓ *Using Two phase method solve,*

$Z_{max} = 5x_1 + 8x_2$

Subject to $3x_1 + 2\ x_2 \geq 3$

$x_1 + 4\ x_2 \geq 4$ and $x_1 + x_2 \leq 5$

Ans: $x_1 = 0, x_2 = 5$ *and* $Z_{max} = 40$

26. *Use Two phase method to solve,*

$Z_{max} = 3x_1 + 2\ x_2 + 3\ x_3$

Subject to $3x_1 + x_2 + x_3 \leq 2$ and $3x_1 + 4\ x_2 + 2\ x_3 \geq 8$

Ans: $x_1 = 0, x_2 = 2, x_3 = 0$ *and* $Z_{max} = 4$

27. $Z_{max} = 3x_1 + 5\ x_2$

Subject to $x_1 \le 4$, $2x_2 \le 12$

$3x_1 + 2x_2 = 18$ and $x_1, x_2 \ge 0$

Ans: *$x_1 = 2$, $x_2 = 6$ and $Z_{max} = 36$*

28. *Minimize $Z = 20x_1 + 10x_2$*

Subject to $x_1 + 2x_2 \le 40$

$3x_1 + x_2 \ge 30$

$4x_1 + 3x_2 \ge 60$ and $x_1, x_2 \ge 0$.

Ans: *$x_1 = 6$, $x_2 = 12$ and $Z_{min} = 240$*

29. *Use penalty method to solve the following LPP*

Maximize $Z = 5x_1 + 3x_2$

Subject to $2x_1 + 4x_2 \le 12$,

$2x_1 + 2x_2 = 10$, $5x_1 + 2x_2 \ge 10$ x_1 and $x_2 \ge 0$

Ans: *$x_1 = 4$, $x_2 = 1$, $Z_{max} = -23$, $Z_{min} = -(Z_{max}) = 23$*

30. *Solve the following LPP by big – M method*

$Z_{max} = -2x_1 - x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \ge 6$

$x_1 + 2x_2 \le 4$ and $x_1, x_2 \ge 0$

Ans: *$x_1 = 3/5$, $x_2 = 6/5$ and $Z_{max} = -12/5$*

31. *Use Big–M method to maximize*

$Z_{max} = 5x_1 + 3x_2$

Subject to $2x_1 + x_2 \le 1$, $x_1 + 4x_2 \ge 6$ and $x_1, x_2 \ge 0$

Ans: *No feasible solution*

32. *Solve $Z_{max} = x_1 + x_2$*

Subject to the constraints $3x_1 + 2x_2 \le 6$, $x_1 + 4x_2 \le 4$ and $x_1, x_2 \ge 0$

Ans: $x_1 = \frac{8}{5}$, $x_2 = \frac{3}{5}$ and $Z_{max} = \frac{11}{5}$

33. *Solve the following LPP*

$Z_{max} = 3x_1 + 5x_2$

Subject to the constraints $2\ x_1 \leq 4$, $3\ x_2 \leq 6$ and $3\ x_1 + 2\ x_2 \leq 18$

Ans: *$x_1 = 2$, $x_2 = 2$ and $Z_{max} = 16$*

34. ***Solve $Z_{max} = x_1 + 2\ x_2$***

Subject to the constraints $x_1 + x_2 \leq 3$, $x_1 + 2\ x_2 \leq 5$ and $3\ x_1 + x_2 \leq 6$

and $x_1, x_2 \geq 0$.

Ans: *$x_1 = 2$, $x_2 = 3$ and $Z_{max} = 5$*

35. ***Solve $Z_{max} = 6\ x_1 - 2\ x_2 + 3\ x_2$***

Subject to the constraints

$$2\ x_1 - x_2 + 2\ x_3 \leq 2$$

$$x_1 + 4\ x_3 \leq 4.$$

Ans: *$x_1 = 4$, $x_2 = 6$, $x_3 = 0$ and $Z_{max} = 12$*

36. ***Solve the following LPP***

$Z_{max} = 2x_1 + x_2$

Subject to $3x_1 + 4x_2 \leq 6$, $6x_1 + x_2 \leq 3$ and $x_1, x_2 \geq 0$

Ans: *$x_1 = 2/7$, $x_2 = 9/7$ and $Z_{max} = 13/7$*

37. ***Solve $Z_{min} = x_1 + x_2$***

Subject to the constraints $x_1 + 2\ x_2 \geq 7$ and $4\ x_1 + x_2 \geq 6$

Ans: $x_1 = \frac{5}{7}$, $x_2 = \frac{22}{7}$ and $Z_{max} = \frac{27}{7}$